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Effect of Axial Variation of Viscosity on flow parameters of blood through a multistenosed artery

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Abstract:
The present study is a mathematical model of the blood flow through a multistenosed artery. The aim of this paper is to analyze the effect of multiple stenosis on the blood flow parameters such as flow rate, flow resistance and shear stress. The rheology of the flowing blood is characterized by the Herchel-Bulkley fluid model with axial variation of viscosity to get the mathematical expression for flow parameters. Blood is taken as non-Newtonian fluid and the flow of blood is considered as steady, laminar, incompressible and one dimensional. The paper is solved analytically using the boundary conditions and the no slip condition on the wall. All the results are plotted by MATLAB software and describes the behaviour of flow parameters. Graph shows that flow rate decreases with increasing the size of stenosis whereas flow resistance and wall shear stress increases with increment in the stenosis size.

Keywords- Herschel-Bulkley fluid model; Multiple Stenosis; Variable Viscosity; Flow Rate; Flow Resistance; Shear Stress.

I.INTRODUCTION

As we know that at present time most of the population is suffering from cardiovascular disease. Cardiovascular disease continues to be the leading cause of death. The development of many arterial disease leading to the serious circulatory problem. The most common arterial disease is stenosis. Stenosis forms due to the deposition of plaque or cholesterol at the various location in the artery. The formation of stenosis disturbs the normal blood flow. The study of blood flow through a stenosed artery is important because the nature of blood movement and mechanical behaviour of vessel walls are cause of many cardiovascular diseases. Various studies have been done earlier for stenosed artery. Varsney et al. [3] investigated the effect of magnetic field on the blood flow in artery having multiple stenosis using the finite difference method. Srikanth and Taddesse [2] analyzed the blood flow through multiple stenotic artery in the presence of catheter assuming blood as incompressible and non-Newtonian. Tiwari and Chauhan [1] presented a mathematical model which deals with viscosity variation through constricted blood vessels, two fluid model is taken into account. Ikbal [6] investigated the viscoelastic blood flow through artery having cosine shaped stenosis in the presence of magnetic field. Mukhopadhyay et al. [8] conducted an investigation to see the impact of viscosity variation on blood flow through an overlapping doubly constricted tapered artery. Gupta et al. [5] studied the impact of radial variation of viscosity and presence of radially non-symmetric stenosis on blood flow in an artery. Gujral and Singh [4] investigated the effect on flow parameters of blood in overlapping atherosclerotic artery considering axial variation of viscosity blood as laminar, steady and axially symmetric using Herschel-Bulkley fluid model.

Here an effort is attempt to explore to study the effect of axial variation of viscosity through a multistenosed artery to evaluate the flow parameters such as flow rate, flow resistance and shear stress.

II. FORMULATION OF THE PROBLEM

Here we have considered the flow of blood through a multistenosed artery, to be laminar, steady and axially symmetric, incompressible, non-Newtonian and using Herschel Bulkley fluid model with axial variation of viscosity ,the expressions for flow parameters such as flow rate, flow resistance and shear stress are evaluated. The study is done for steady, laminar, fully developed flow, which is symmetric about the axis and one-dimensional. The geometry of the arterial segment having the multiple stenosis mathematically is given by [7]

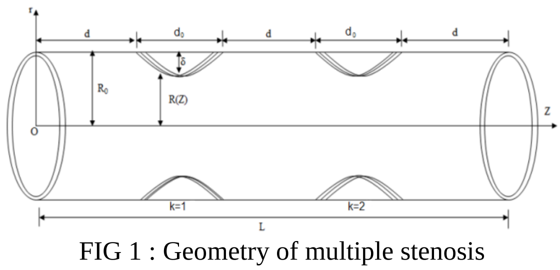

FIG 1 : Geometry of multiple stenosis

$$\frac{R(z)}{R_0} = \begin{cases} 1 - A[d_0^{s-1}(az - kd - (k-1)d_0) - (az - kd - (k-1)d_0)^s] \\ \qquad ; k(d + d_0) - d_0 \le az \le k(d + d_0) \\ 1 \qquad ; \ otherwise \end{cases}$$

where, $h = \frac{\delta}{R_0}$ and $A = \frac{h}{d_0^s}\frac{s^{s/(s-1)}}{(s-1)}$

$\delta (<< R_0)$ is the maximum height of the stenosis.

$Z = \frac{1}{a}\left\{kd + (k-1)d_0 + \frac{d_0}{s^{1/(s-1)}}\right\}$

Where R(z) and R_0 are the constricted and unconstricted radius of the artery respectively. Let L be the length of the artery, d_0 is te length of the stenosis, d is distance between equi spaced point, k is number of the stenosis, a (≥ 1) is a parameter and s (≥ 2) is the shape parameter.

The constitutive equation for Herschel-Bulkley fluid is given by [4],

$$\left(-\frac{\partial u}{\partial r}\right) = \begin{cases} \frac{1}{\mu(z)}(\tau-\tau_0)^n & ,\tau \geq \tau_0 \\ 0 & ,\tau < \tau_0 \end{cases} \quad \text{----------(2.1)}$$

Where u is the axial velocity of the blood, τ is the shear stress, τ_0 is the yield shear stress, $\mu(z)$ is the viscosity of the fluid in the axial direction and n is the fluid behaviour index.

Viscosity in the axial direction is given by,

$$\mu(z) = \begin{cases} \mu_0\left(\frac{R(z)}{R_0}\right)^{-\alpha} & ;\ n(d+l_0)-l_0 \leq az \leq n(d+l_0) \\ \mu_0 & ;otherwise \end{cases} \quad \text{---------------(2.2)}$$

where α is the parameter and μ_0 is the plasma viscosity.

The boundary conditions are as follows,

$\frac{\partial u}{\partial r} = 0$ at r = 0 -------------(2.3a)
u = 0 at r = R(z) -------------(2.3b)
τ *is* finite at r = 0 -------------(2.3c)
P = P_0 at z = 0 and P = P_L at z = L -------------(2.3d)

Navier -Stokes equation in cylindrical coordinate system for blood is given by,

$$0 = -\frac{\partial p}{\partial z} - \frac{1}{r}\frac{\partial (r\tau)}{\partial r} \quad \text{-------------(2.4a)}$$

$$0 = \frac{\partial p}{\partial r} \quad \text{-------------(2.4b)}$$

III. SOLUTION OF THE PROBLEM

Substituting the value of τ from equation (2.1) in the equation (2.4a), we get

$$0 = -\frac{\partial p}{\partial z} - \frac{1}{r}\frac{\partial}{\partial r}\left[r\left\{\mu(z)^{\frac{1}{n}}\left(-\frac{\partial u}{\partial r}\right)^{\frac{1}{n}} + \tau_0\right\}\right] \quad \text{------------(3.1)}$$

Integrating equation (3.1) with respect to r we get,

$$\left(-\frac{\partial u}{\partial r}\right)^{\frac{1}{n}} = \frac{1}{\mu(z)^{\frac{1}{n}}}\left[\frac{r}{2}\left(-\frac{\partial p}{\partial z}\right) - \tau_0\right] \quad \text{--------------(3.2)}$$

The constant flux is given by,

$$Q = \int_0^{R(z)} 2\pi r u dr = \int_0^{R(z)} \pi r^2\left(-\frac{\partial u}{\partial r}\right) dr$$

$$Q = \left(-\frac{1}{2}\frac{\partial p}{\partial z}\right)^n \frac{\pi}{\mu(z)} \mathrm{I}(R(z)) \quad \text{--------------(3.3)}$$

where,

$$\mathrm{I}(R(z)) = \int_0^{R(z)}\left[r + \frac{2\tau_0}{\frac{\partial p}{\partial z}}\right]^n dr$$

From (3.3), $\left(-\frac{1}{2}\frac{\partial p}{\partial z}\right) = 2\left(\frac{Q\mu(z)}{\pi \mathrm{I}(R(z))}\right)^{\frac{1}{n}}$ ---------------(3.4)

For solving pressure drop ∇P, integrating (3.4) and using boundary condition (2.4d),

$$\nabla P = -(P_L - P_0) = 2\left(\frac{Q}{\pi}\right)^{\frac{1}{n}}\int_0^L\left(\frac{\mu(z)}{I(R(z))}\right)^{\frac{1}{n}} dz \quad \text{------------(3.5)}$$

Flow resistance is given by,

$$\lambda = \frac{\nabla P}{Q} = 2\left(\frac{Q^{1-n}}{\pi}\right)^{\frac{1}{n}}\int_0^L\left(\frac{\mu(z)}{(R(z))}\right)^{\frac{1}{n}} dz \quad \text{----------------(3.6)}$$

Shear stress is given by,

$$\tau = \tau_0 + \left[-\frac{\partial u}{\partial r}\mu(z)\right]^{\frac{1}{n}} = r\left(\frac{Q\mu(z)}{\pi \mathrm{I}(R(z))}\right)^{\frac{1}{n}} \quad \text{----------------(3.7)}$$

At r = R(z) Shear stress is given by,

$$\tau = \mathrm{R}(z)\left(\frac{Q\mu(z)}{\pi \mathrm{I}(R(z))}\right)^{\frac{1}{n}}$$

For solving this problem, we consider two cases of viscosity variation.

<u>Case 1</u>: Consider $\alpha = 1$ in equation (2.2) i.e., linear variation of viscosity, then

$$\mu(z) = \mu_0\left(\frac{R(z)}{R_0}\right)^{-1}$$

substituting the value of $\mu(z)$ in equation (3.5),
Then the Flow rate is given by,

$$Q = \left(-\frac{1}{2}\frac{\partial p}{\partial z}\right)^n \frac{\pi}{\mu_0}\frac{R(z)}{R_0} I(R(z)) \quad \text{-----------------(3.8)}$$

substituting the value of $\mu(z)$ in equation (3.6),
Flow resistance is given by,

$$\lambda = \frac{\nabla P}{Q} = 2\left(\frac{\mu_0 Q^{1-n}}{\pi}\right)^{\frac{1}{n}}\int_0^L\left(\frac{1}{I(R(z))}\frac{R_0}{R(z)}\right)^{\frac{1}{n}} dz \quad \text{-------------(3.9a)}$$

$$\lambda = 2\left(\frac{\mu_0 Q^{1-n}}{\pi}\right)^{\frac{1}{n}}\left[\int_0^{k(d+d_0)-d_0}\left(\frac{1}{I(R(z))}\frac{R_0}{R(z)}\right)^{\frac{1}{n}} dz + \int_{k(d+d_0)-d_0}^{k(d+d_0)}\left(\frac{1}{I(R(z))}\frac{R_0}{R(z)}\right)^{\frac{1}{n}} dz + \int_{k(d+d_0)}^{L}\left(\frac{1}{I(R(z))}\frac{R_0}{R(z)}\right)^{\frac{1}{n}} dz\right]$$

At R(z) = R_0, Flow resistance at the wall is given by,

$$\lambda_w = 2\left(\frac{\mu_0 Q^{1-n}}{\pi}\right)^{\frac{1}{n}}\int_0^L\left(\frac{1}{I_0(R_0)}\right)^{\frac{1}{n}} dz \quad \text{----------------(3.9b)}$$

where,

$$I_0(R_0) = \int_0^{R_0}\left[r + \frac{2\tau_0}{\frac{\partial p}{\partial z}}\right]^n dr$$

Then λ^- is given by,

$$\lambda^- = \frac{\lambda}{\lambda_w} = \frac{\int_0^L\left(\frac{1}{I(R(z))}\frac{R_0}{R(z)}\right)^{\frac{1}{n}} dz}{\int_0^L\left(\frac{1}{I_0(R_0)}\right)^{\frac{1}{n}} dz} \quad \text{--------------(3.10)}$$

substituting the value of $\mu(z)$ in equation (3.7),

Shear stress for linear variation of viscosity

$$\tau = R(z)\left(\frac{Q\mu_0}{\pi I(R(z))}\frac{R_0}{R(z)}\right)^{\frac{1}{n}} \quad \text{-----------------(3.11a)}$$

At R(z) = R_0, Shear stress at the wall is given by,

$$\tau_w = R(z)\left(\frac{Q\mu_0}{\pi I(R_0)}\right)^{\frac{1}{n}} \quad \text{------------------ (3.11b)}$$

Then τ^- is given by,

$$\tau^- = \frac{\tau}{\tau_w} = \left(\frac{R(z)}{R_0}\right)^{\frac{n-1}{n}}\left(\frac{I_0(R_0)}{I(R(z))}\right)^{\frac{1}{n}} \quad \text{------------- (3.12)}$$

Case 2: Consider α = 2 in equation (2.2), i.e., quadratic variation of viscosity, then

$$\mu(z) = \mu_0\left(\frac{R(z)}{R_0}\right)^{-2}$$

substituting the value of $\mu(z)$ in equation (3.5),
Then the Flow rate is given by,

$$Q = \left(-\frac{1}{2}\frac{\partial p}{\partial z}\right)^n \frac{\pi}{\mu_0}\left(\frac{R(z)}{R_0}\right)^2 I(R(z)) \quad \text{------------(3.13)}$$

substituting the value of $\mu(z)$ in equation (3.6),
Flow resistance is given by,

$$\lambda = \frac{\nabla P}{Q} = 2\left(\frac{\mu_0 Q^{1-n}}{\pi}\right)^{\frac{1}{n}}\int_0^L\left(\frac{1}{I(R(z))}\left(\frac{R_0}{R(z)}\right)^2\right)^{\frac{1}{n}}dz \quad \text{------(3.14a)}$$

$$\lambda = 2\left(\frac{\mu_0 Q^{1-n}}{\pi}\right)^{\frac{1}{n}}\left[\int_0^{k(d+d_0)-d_0}\left(\frac{1}{I(R(z))}\left(\frac{R_0}{R(z)}\right)^2\right)^{\frac{1}{n}}dz + \int_{k(d+d_0)-d_0}^{k(d+d_0)}\left(\frac{1}{I(R(z))}\left(\frac{R_0}{R(z)}\right)^2\right)^{\frac{1}{n}}dz + \int_{k(d+d_0)}^{L}\left(\frac{1}{I(R(z))}\left(\frac{R_0}{R(z)}\right)^2\right)^{\frac{1}{n}}dz\right]$$

At R(z) = R_0, Flow resistance at the wall is given by,

$$\lambda_w = 2\left(\frac{\mu_0 Q^{1-n}}{\pi}\right)^{\frac{1}{n}}\int_0^L\left(\frac{1}{I_0(R_0)}\right)^{\frac{1}{n}}dz \quad \text{------------------(3.14b)}$$

Then λ^- is given by,

$$\lambda^- = \frac{\lambda}{\lambda_w} = \frac{\int_0^L\left(\frac{1}{I(R(z))}\left(\frac{R_0}{R(z)}\right)^2\right)^{\frac{1}{n}}dz}{\int_0^L\left(\frac{1}{I_0(R_0)}\right)^{\frac{1}{n}}dz} \quad \text{-----------------(3.15)}$$

substituting the value of $\mu(z)$ in equation (3.7),
Shear stress for quadratic variation of viscosity,

$$\tau = R(z)\left(\frac{Q\mu_0}{\pi I(R(z))}\left(\frac{R_0}{R(z)}\right)^2\right)^{\frac{1}{n}} \quad \text{-------------------(3.16a)}$$

At R(z) = R_0, Shear stress at the wall is given by,

$$\tau_w = R(z)\left(\frac{Q\mu_0}{\pi I(R_0)}\right)^{\frac{1}{n}} \quad \text{-------------------(3.16b)}$$

Then τ^- is given by,

$$\tau^- = \frac{\tau}{\tau_w} = \left(\frac{R(z)}{R_0}\right)^{\frac{n-2}{n}}\left(\frac{I_0(R_0)}{I(R(z))}\right)^{\frac{1}{n}} \quad \text{-------------------(3.17)}$$

IV. RESULT AND DISCUSSION

In this section, the numerical results for flow parameters such as flow rate, resistance to flow and wall shear stress are presented in the presence of multiple stenosis considering axial variation of viscosity. The problem is solved analytically and all results are shown graphically using MATLAB software.

In FIG 2(a) and 2(b), the graph has been plotted between the flow rate and the axis of the artery varying the fluid behavior index and yield stress respectively. The graph shows that while moving along the axis flow rate first decreases, after reaching at the maximum height of stenosis it reaches to the minimum and then come back to its initial state. Due to presence of the multiple stenosis again the flow decreases and returns to initial flow. In FIG 2(c), the graph shows the comparison of flow rate between linear and quadratic variation of the viscosity. It is observed that flow rate in case of linear variation is more as compared to the quadratic variation of viscosity.

In FIG 3(a) and 3(b), graph shows the variation of flow rate with the size of the stenosis varying fluid behavior index and yield stress respectively. It is observed that as the size of stenosis is increased, flow rate is decreased with the increase in the value of n and τ_0. In FIG 3(c), the graph shows the comparison of flow rate between linear and quadratic variation of the viscosity. It is observed that flow rate in case of linear variation is more as compared to the quadratic variation of viscosity.

In FIG 4(a) and 4(b) graph have been plotted for the flow resistance and size of the stenosis varying n and τ_0 respectively. It is noticed that flow resistance increases as size of the stenosis increases. And as the value of n and τ_0 increases, flow resistance decreases. In FIG 4(c), the graph shows the comparison of flow resistance between linear and quadratic variation of the viscosity. It is observed that flow resistance in case of linear variation is more as compared to the quadratic variation of viscosity.

In FIG 5(a) and 5(b), the graph have been plotted between shear stress and size of the stenosis for the distinct values of n and τ_0. It is noticed that wall shear stress increases as the size of stenosis increases. It is also noticed that with increase in fluid behavior index n, wall shear stress increases and with yield stress τ_0,wall shear stress decreases. In FIG 5(c), the graph shows the comparison of wall shear stress between linear and quadratic variation of the viscosity. It is observed that wall shear stress in case of quadratic variation is more as compared to the linear variation of viscosity.

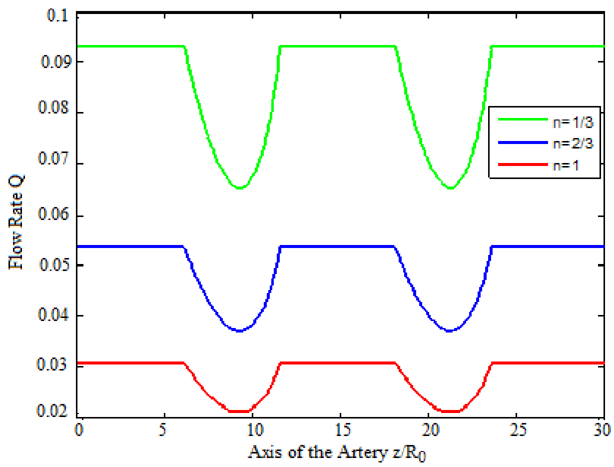

FIG 2(a): Graph of flow rate versus axis of the artery varying fluid behavior index n

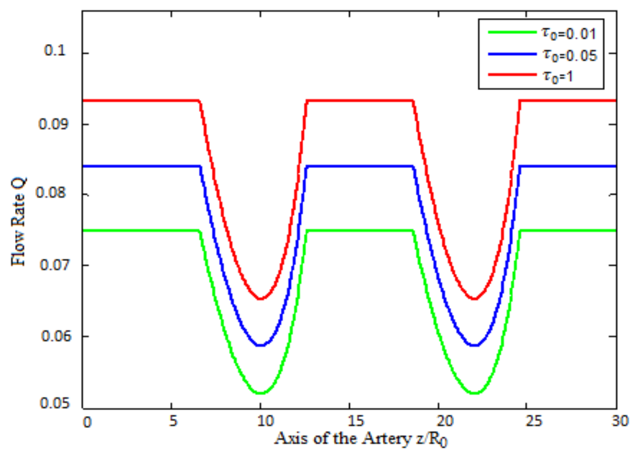

FIG 2(b): Graph of flow rate versus axis of the artery yield stress τ_0

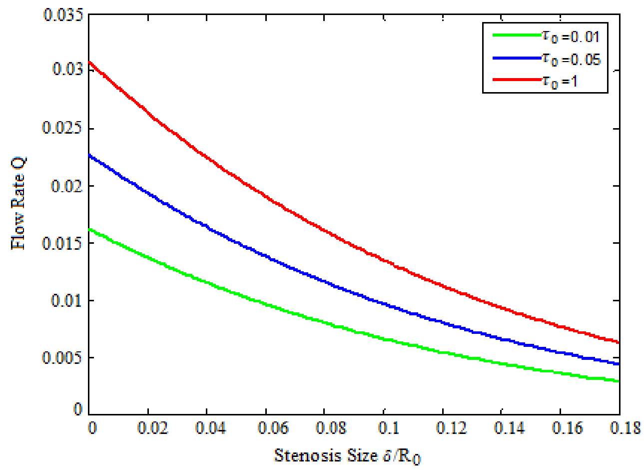

FIG 3(b): Graph of flow rate versus size of the stenosis varying yield stress τ_0

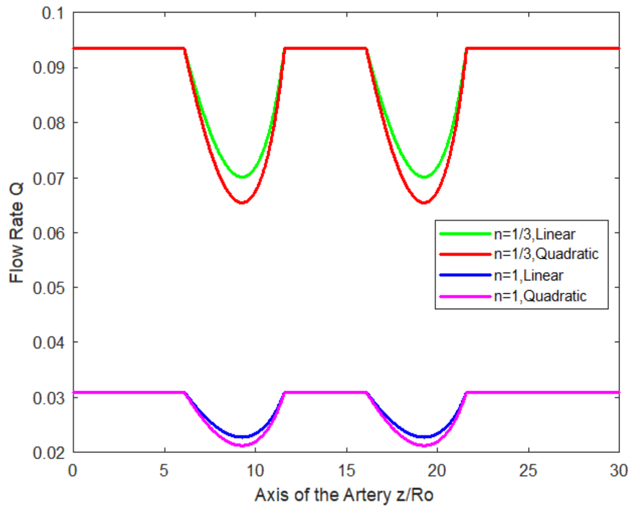

FIG 2(c): Comparison of flow rate between linear and quadratic variation of the viscosity.

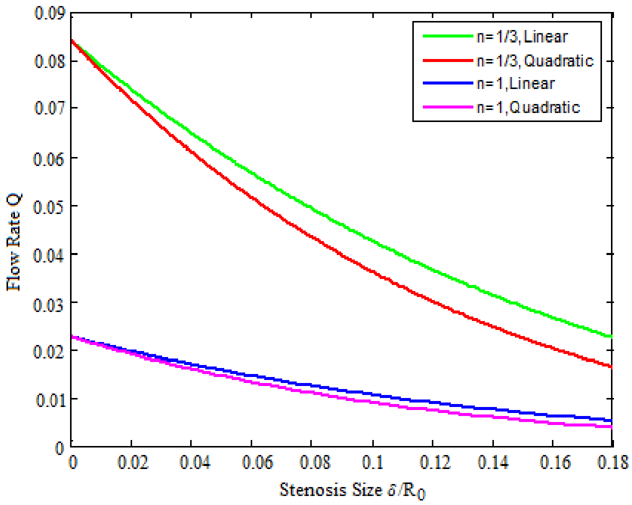

FIG 3(c): Comparison of flow rate between linear and quadratic variation of the viscosity varying the fluid behavior index n

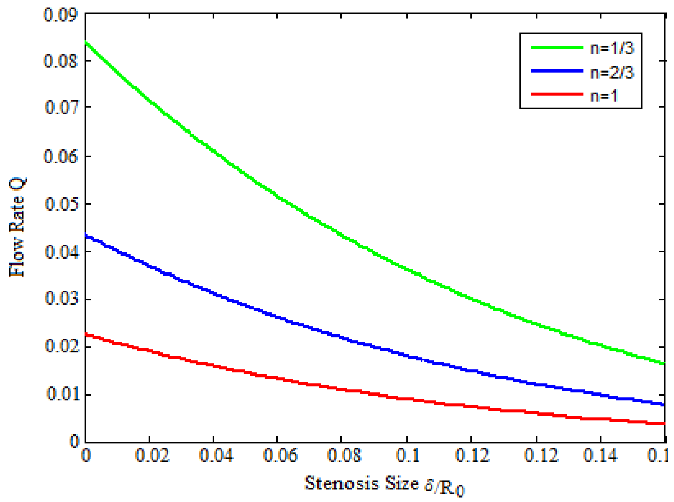

FIG 3(a): Graph of flow rate versus size of the stenosis varying fluid behavior index n

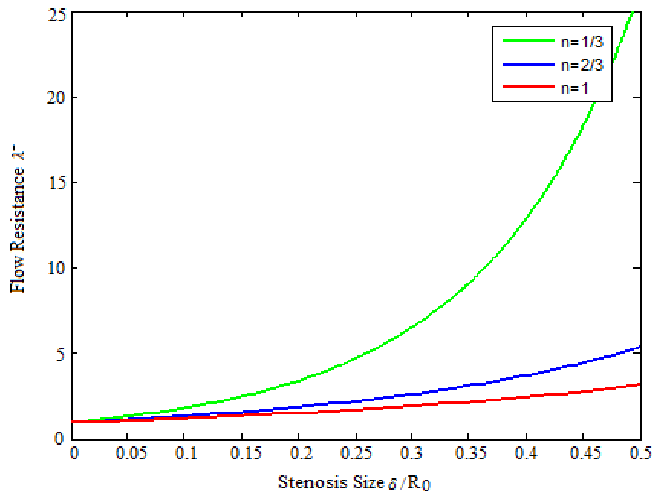

FIG 4(a): Graph of flow resistance versus size of the stenosis varying fluid behavior index n

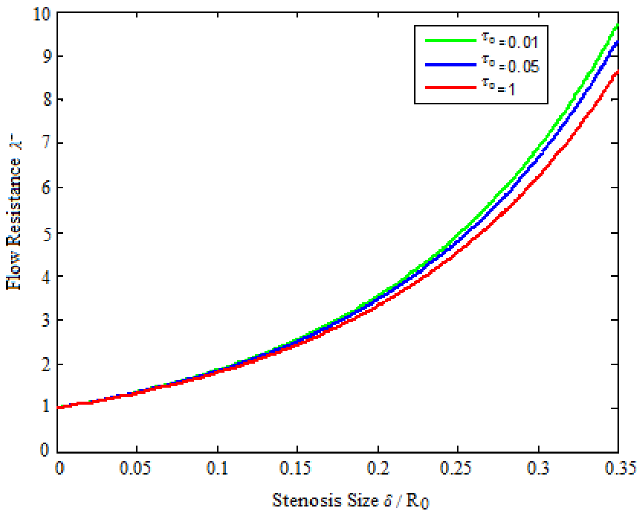

FIG 4(b): Graph of flow resistance versus size of the stenosis varying yield stress τ_0

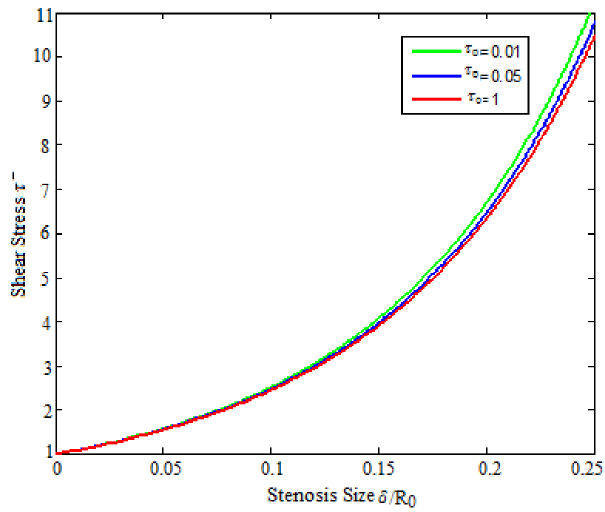

FIG 5(b): Graph of flow wall shear stress versus size of the stenosis varying yield stress τ_0

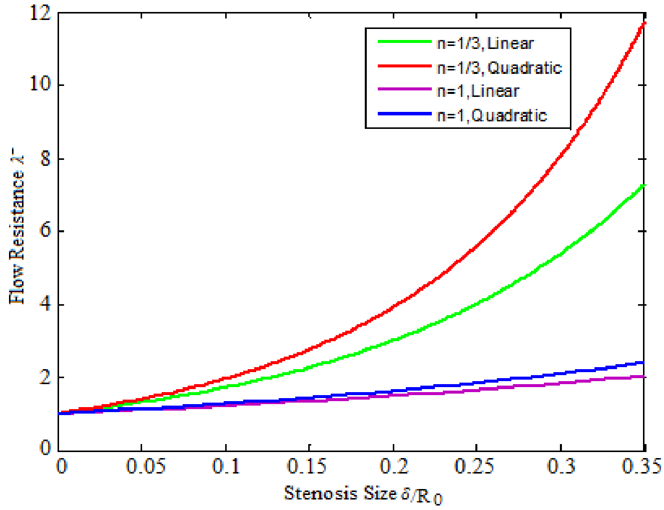

FIG 4(c): Comparison of flow resistance between linear and quadratic variation of the viscosity varying the fluid behavior index n

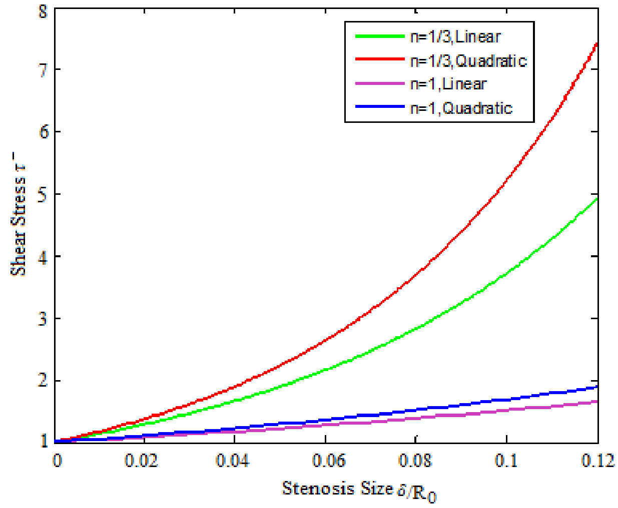

FIG 5(c): Comparison of shear stress between linear and quadratic variation of the viscosity varying the fluid behavior index n

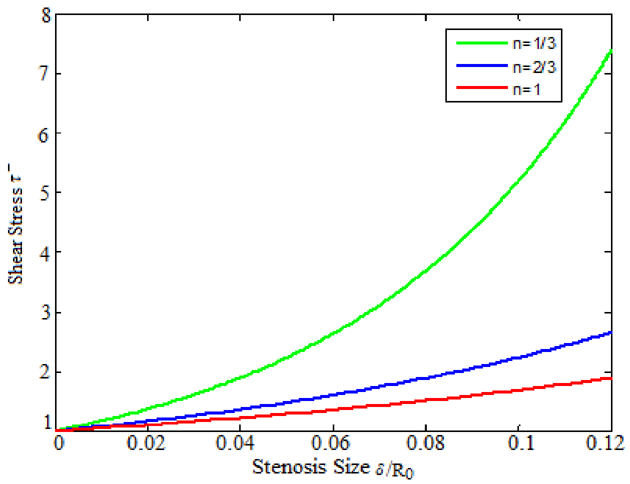

FIG 5(a): Graph of flow wall shear stress versus size of the stenosis varying fluid behavior index n

V. CONCLUSION

This section of the paper devoted to the influence of linear and quadratic variation of viscosity of blood in the presence of multiple stenosis. Expressions are evaluated for the flow parameters such as flow rate, flow resistance and wall shear stress. Results are discussed with the help of graphs. It can be seen flow rate decreases as the size of the stenosis increases while flow resistance and wall shear stress increases on increasing the size of the stenosis and further it is also noticed that flow rate in case of linear variation of viscosity has slightly greater values as compared to the case of quadratic variation of viscosity where as flow resistance and wall shear stress in case of linear variation of viscosity has slightly less values as compared to the case of quadratic variation of viscosity.

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