Some studies on Simple Weak Alternative Novikov Rings

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Abstract- In this paper we studied Associative of a ring on an alternative ring with an idempotent E * 0, when the condition $R_y R_{jj}=R_u$ and verification of a ring is mandatorily associative on a semiprime alternative ring that satisfies the weak identity of Novikov y(w,x,z), = (w,x,yz). Also Verified ring is associative if R is alternative satisfies weak Novikov identity and is semiprime on Simple Weak Alternative Novikov Rings.
Key words: Alternative rings, Semi prime, Novikov identity

I. INTRODUCTION

Simple finite dimensional strongly Novikov algebras over a field has been classified by E.I.Zelmanov [1] and the rings are both associative and commutative. Kleinfeld and Smith generalised this result to weakly Novikov rings [2]. The weak Novikov identity is satisfied through the Right alternative asy(w,x,z) = (w,x,yz) [3]. Also, it was validated that weak Novikov rings which are semiprime flexible are associative [4], This result complements the study of right alternative rings that satisfy the weak Novikov identity. So that strongly Novikov rings are not aassociative rings subclass. Since (wx)(yz) - w(x.yz) = (w,x,yz) = y(w,x,z) = y(wx.z) - y(w.xz), strongly Novikov rings are weakly Novikov.

II.PROPOSED THEOREMS

Kleinfeld studied and proved that a rings with simple alternative property is either Associative or Cayley-Dickson algebra. In simple alternative rings we can see that fourth powers of commutators are in the nucleus [5]. Kleinfeld and Smith [6] proved that when is R is associative simple right alternative ring which is 2-divisible has the commutators in the left nucleus.

Simple Right Alternative Rings :
"Right alternative rings were first studied by A.A. Albert, who showed that a semi-simple right alternative algebra over a field is alternative" [7],
In 2- divisible rings with right alternative the following identity is hold:

(w, y, (z,x)) + (wy, z, x) = (w, z, x)y +. w (y, z, x)……. 1

In arbitrary rings the two identities hold:
(w, x, y)z + w(x, y, z) = (wx, y, z) - (w, xy, z) + (w, x, yz)…….2

(z,x,y) + (x,y,z) + (y,z,x) =(zx,y) + (xy, z) + (yz,x) 3

Lemma 1: $\sum R + (R, R) = R$

Proof :
I = (R,R)R + I (R,R) is a two-sided ideal, using the subset of Nt in the form of (R,R). The ideal is a non-zero, when the ring R is non commutative, simple and equal to I
If n N. Replace w with n in 2. yields

n(x,y,z))=(nx.y.z)................... 4

Since (y,z (n,x),) = 0 we get

(nx,y,z) = (xn,y,z) = n(x,y,z) 5

Assume p,q N_i. Then p (xq, y, z) = p.q (x, y, z) = pq.(x, y, z)
(px.q, y, z) = (p. xq, y, z) = (px.y.z) q = (x, y, z) q. p =(x, y, z)qp.
using 5 and the Nt definition
(x, y, z) (p, q) = 0, or

(N, N_i) (R,R,R,) = 0. 6

The ideal of associator A = (R,R,R)R + R (R,R,R). Since (N, N_i) N which obtained from 5
(N) A (N)t = 0,with R is non associative and simple, A=R≠ 0. R is simple,

(N, N_i) R = 0. 7

From 1.it was obtained that
(a,b,c) (x, y) + a ((x,y), b,c) = (a, (x,y), (b,c)) + (a (x,y), b,c)
However, (b, c, (x, y),) = 0 and 4 implies
(b, c, a (x, y)) = (b, c, a) (x, y)
((a, b, c,) (x, y)) = - ((b,c), a, (x, y)) 8

Because of right alternativity,

- (a, (x, y), (b,c)} = (a, (b,c), (x, y)).................. 9

Applying the same permutation to 8 is given as,

((a, b, c), (x, y)) = - ((a, x, y), (b, c),) 10

Now substitute x = p N_i and y = q N_i in 10.

{(a, b, c), (p,q)) = - ((a, p, q), (b, c)) 11

It was found that,
(a, p, q) =(p, q, a)+ (a, p, q) + (q, a, p) =(pq, a) +(ap, q) + (qa, p) and substituting it in 11
((p.q), (a.b.c)) = - ((ap, q), (b, c)+ (qa, p) + (pq, a)).
Let additive group J be generated with the elements from (N_i, N),
Thus
((a, b, c), (p, q)) J. From 5, (a, b, c) (p, q) = 0, sothat

(N, N) (R, R, R) J. 12

From 6 (R, R) (N,N) = 0. Thus

(N, N) (R, R) = {(N,N) (R, R)} (N, N,) J or
(Ni, N,)(R, R) J....................... 13

Lemma 2: Let R be an alternative ring with an idempotent E * 0, when the condition R_y R_{jj}=R_u is satisfied then the corresponding ring R is associative

Proof : R_{io} = (e,R_{io}) = -(R_{io}.e) and Roi = (R_{oi},e). Since E N, using lemma 2,
Both R_{io}, R_{oi} N, where N is R based associative subring.
So $R_{io}R_{oi}$and R_{oi} R_{io} N. By the given condition R_n and R_{oo} N. It follows that R = R_{io}+ R_{oo}+ R_{oi} + R_n N,
Thus, R is associative ring.

Theorem 1: If R is a semiprime alternative ring that satisfies the weak identity of Novikov y(w,x,z), = (w,x,yz) then R is mandatorily associative.

Proof : when the ring is alternative, then
(y,z,x^2) = (x^2,y,z) = (y,z,x)x+x(y,z,x).
It was also observed that x(y,z,x)= (y,z,x^2). For satisfying this from both

(y,z,x)x and (x,y,z)x = 0......... 14 and it as the alternative identities as

(x,y,z)x =(x,y,xz)= (x^2,y,z) = x(x,y,z)= 0
For achieving this condition, x^2 should be in N

x^2 N....... 15
when linearizing is applied over 15 and considering both x and y $\in R$, we obtain
yx+xy N........... 16

For all x,y g R. However n(x,y,z) =(x,y,nz), and N is a R ideal, then

NI = 0. 17

But R is semiprime implies N n I = 0, ... 18

Also (x,y,z)w + w(x,y,z) N from 16 and it belongs to I. From 18 it follows that

(x,y,z)w + w(x,y,z)= 0, ... 19

for all variables in 19 are taken from R.
Then, u g
u(x,y,z).w + w.u(x,y,z) = (x,y,uz)w + w(x,y,uz)= 0, using 19
Thusanti-commutes for w gives,

u(x,y,z), ... 20

Now consider (x,wy,vz). Using twice, this implies v.w(x,y,z) =w. v(x,y,z)... 21

Integrating 20 and 21 it was obtained as w(x,y,z).u= u(x,y,z).w...... 22

However from 19 we have - (x,y,z)w = w(x,y,z) .
Substituting 22

- (x,y,z)w.u= u(x,y,z).w......... 23

With anticommutation of u over (x,y,z) w from 20 result in w (x,y,z).
Applying 19 in 23

w.(x,y,z)u= u(x,y,z).w .... 24 and it forms into

(u,(x,y,z),w) = 0. 25

Hence (R,R,R) N_nI. 26

But from 18, N_oI = 0 = (R,R,R)
Thus, it was proved that R is associative and it is the proof of theorem.
The ring R that satisfied the identity 0= (x,y,y) is denoted as the right alternative ring and it also satisfied the weak identity of Novikov as,
y(w, x, z) = (w, x, yz)
The commutative centre C and right nucleus (R_n) of the ring R is given as
R_n= {n R / (R, R, n) = 0 } and C= {c R/(R, c) = 0 }.
Let A be the ideal of associator that contains all finite associator sums and left associator multiples. From 3 the ring R with associator ideal may be given as all finite associators sums.
Lemma 3: If R is alternative satisfies weak Novikov identity and is semiprime then R is associative.
Proof: In an identity of alternative ring

(x^2, y, z) = (y,z,x)x + x(y, z, x) = (y, z, x^2) [35]. On the other hand it is observed that x (y,z,x)= (y, z, x^2) similarly (x, y, z)x = 0, so that

(y, z, x)x = 0..................... 27

"At this point we have the alternative identity 0 = (x, y, z) x = (x, y, xz) [35], using 27 we get
(x^2, y, z) = 0. Let N be the nucleus of R. we have shown that
x^2 N, for all x   R". 28

Linearizing 28, we obtain

xy + yx   N, for all x, y   R. 29

Since for any alterative ring, N_r = N

N is an ideal of R. 30

NA = 0. 31

With semi-prime hypothesis, 31 can be given as

N n A = 0. 32

From 28 and 32

(x,y,z)w + w (x, y, z) = 0................................ 33

Then for u   R, w. u (x, y, z)w + u (x, y, z) = w (x, y, uz) + (x, y, uz)w = 0, using 33. Thus

w anticommutes with u (x, y, z)..................... 34

now initialize (x, wy, uz) and take u and w from associator in two different order as,

u. w (x, y, z) =w. u (x, y, z)..................... 35

By combining 35 with 34 we obtain

w (x, y, z). u= u (x, y, z). w.................... 36

Substituting 36 in 37

- (x, y, z) w. u =u (x, y, z). w........................... 37

With anticommutation of 34 and 33 with w (x, y, z) and (x, y, z)wrespectively on 37 provided

v. (x, y, z) w = v (x, y, z). w........................... 38

The associator form of 38 is given as

(v, (x, y, z),w) = 0. 39

From 39, the equivalent form can be given as,

(R,R,R) N_n A 40

This showed that R is associative.

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