HYDROMAGNETIC CONVECTIVE HEAT AND MASS TRANSFER FLOW PAST A VERTICAL CORRUGATED SURFACE WITH VARIABLE VISCOSITY, THERMAL CONDUCTIVITY, THERMAL RADIATION, ACTIVATION ENERGY IN THE PRESENCE OF IRREGULAR HEAT SOURCES

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Abstract- We investigate impact of variable viscosity, activation energy on convective heat and mass transfer flow over a vertical wavy surface embedded with thermal radiation and heat sources. The temperature dependent variables properties are considered. The governing equations have been solved by Runge-Kutta Shooting technique. The effect of variable viscosity, activation energy, and thermal radiation on the flow variables has been discussed graphically. It is found that the velocity(u) depreciates, temperature(θ) and concentration(ϕ) increase in the flow region with higher values of viscosity parameter (θr). The velocity, concentration enhance with activation energy E1 and reduces with temperature difference parameter(δ),temperature reduces with E1 and reduces with δ. Increasing thermal conductivity parameter (β) increases temperature and reduces velocity, concentration.

Keywords: Variable viscosity, Thermal conductivity, Activation energy, Thermal Radiation, Corrugated surface, non-uniform heat sources

I. INTRODUCTION

In the combined heat and mass transfer processes the flow is driven by density differences caused by temperature gradient concentration gradient and material compassion simultaneously.

In most of the studies, the viscosity of the fluid was assumed to be constant [4,26].Tthe flow characteristics in case of variable viscosity are significantly change compared to the constant property case. Viscosity and thermal conductivity of fluids vary with temperature in convection flow in nature and engineering phenomena. For instance, the viscosity of dry air at 100^0 C is $21.94\text{x}10^{-6}$ kg/ms while at 200^0 C, it is $26.94\text{x}10^{-6}$ kg/ms. The effects of temperature-dependent viscosity and variable thermal conductivity on mixed convective diffusion flow. Mallikarjuna [9] have discussed the effect of variable viscosity and thermal conductivity on convective heat and mass transfer flow over a vertical wavy surface in a porous medium with variable properties has been analysed by Dulal and Hiramony [5].Several authors (Vajravelu et al [1], Shweta [14] and Isaac and Anselm [6] deliberated that, velocity distribution decreases with increase in viscosity while the temperature profiles increase with increase in variable viscosity and thermal conductivity on hydromagnetic flow.

Mass transfer phenomenon carried out by chemical reaction and activation energy occurs in different fields of science like chemical engineering, geo-thermal reservoirs, oil emulsions, food processing, mechano chemistry. Activation energy is defined as the minimum amount of energy owned by a reacting specie to undergo an indicated reaction. The species with changing concentration in a mixture carry themselves from the state of higher concentration to the state of lower concentration. The Arrhenius equation is usually of the form (Tencer et al.[4])

$$K = B\left(\frac{T}{T_\infty}\right)^n \exp\left(\frac{-Ea}{kT}\right), \quad (1)$$

where K represents reaction rate constant, Ea is the activation energy, T the fluid temperature and B the pre-exceptional factor depicting an increase in the temperature with respect to increase in the reaction rate.

Before 1990, no attention was given to the effect of activation energy on mass flow. After examining the behaviour of binary chemical reaction and activation energy in the MHD viscoelastic fluid flow over linearly stretching sheet, Mustafa et al.,[10] observed that the concentration profile escalates by escalating the value of activation energy. However, in 1990, Bestman [3] worked on the effect of Arrhenius activation energy in the heat and mass transfer in a porous medium. The effect of activation energy on an unsteady free convective flow of heat and mass transfer has been discussed by Abdul Maleque [8].He observed that the velocity profile enhances by increasing the value of the Grashof number.

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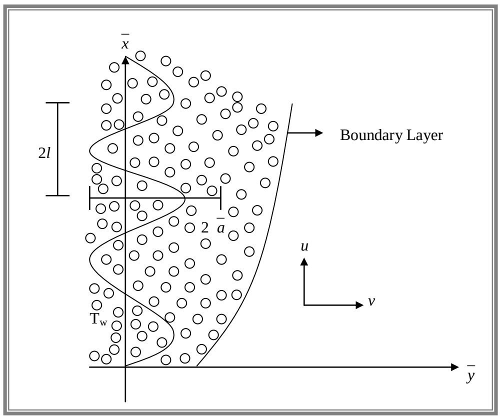

Fig. – 1 : Physical Configuration and Co-ordinate System

In this paper, we investigate combined influence of variable viscosity, thermal conductivity, activation energy on natural convective heat and mass transfer flow over a vertical wavy surface embedded in the presence of heat sources. The temperature dependent variables properties are considered. The vertical wavy wall and the governing equations for flow, heat and mass transfer are transformed to a plane geometry case by employing the Rungre-Kutta fourth order with shooting technique. The non-dimensional velocity, temperature and concentration graphs as well as skin friction, rate of heat and mass transfer coefficients are displayed for different values of variable parameter, variable viscosity, thermal conductivity, activation energy parameter, radiation parameter, and amplitude of the wavy surface. The obtained results are compared with those presented by Mallikarjuna et al [9] and excellent agreement has been reported in the absence of activation energy, heat sources, thermal radiation.

II. FORMULATION OF THE PROBLEM

The steady incompressible two-dimensional laminar natural convective heat and mass transfer flow over a vertical wavy surface is considered. A uniform magnetic field of strength Ho is applied normal to the wall. The fluid is assumed to be gray, absorbing-emitting radiation but non-scattering medium.

The wavy surface profile is given by

$$y = \bar{\sigma}(\bar{x}) = \bar{a} Sin(\frac{\pi \bar{x}}{l}) \tag{1}$$

where l is the characteristic length of wavy surface and $\bar{a}$ is the amplitude of the wavy surface .The wavy surface is maintained at constant temperature Tw which are higher than the ambient fluid temperature T_∞. We consider the natural convection-radiation flow in the presence of heat sources to be governed by the following equations under Boussinesq and Rosseland approximations

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{2}$$

$$\frac{\partial}{\partial \bar{y}}(\frac{\mu}{k}\bar{u}) - (\sigma\mu_e^2 H_o^2)\frac{\partial u}{\partial y} = \frac{\partial}{\partial \bar{x}}(\frac{\mu}{k}\bar{v}) + \rho g(\beta_T \frac{\partial T}{\partial y} + \beta_c \frac{\partial C}{\partial y}) \tag{3}$$

$$\bar{u}\frac{\partial T}{\partial \bar{x}} + \bar{v}\frac{\partial T}{\partial \bar{y}} = \frac{\partial}{\partial \bar{x}}(\alpha \frac{\partial T}{\partial \bar{x}}) + \frac{\partial}{\partial \bar{y}}(\alpha \frac{\partial T}{\partial \bar{y}}) + \frac{kRa}{L^2 \rho \xi C_p}(A'_{11}(T_w - T_\infty) + B'_{11}(T - T_\infty)) + \frac{1}{C_p}(\frac{16\sigma^{\bullet} T_\infty^3}{3\beta_R})(\frac{\partial^2 T}{\partial \bar{x}^2} + \frac{\partial^2 T}{\partial \bar{y}}^2) \tag{4}$$

$$u\frac{\partial C}{\partial \bar{x}} + v\frac{\partial C}{\partial \bar{y}} = D_b(\frac{\partial^2 C}{\partial \bar{x}^2} + \frac{\partial^2 C}{\partial \bar{y}^2}) - k_c^{'}(\frac{T}{T_\infty})^n (C - C_\infty) Exp(-\frac{E_n}{kT}) \tag{5}$$

The relevant boundary conditions are

$$\bar{u} = 0, \bar{v} = 0, T = T_w \ , C = C_w \ at \ \bar{y} = \bar{\sigma}(\bar{x}) = \bar{a}\ Sin(\frac{\pi \bar{x}}{l})$$

$$\bar{u} \to 0, T \to T_\infty \ , C \to C_\infty \ as \ \ \bar{y} \to \infty \tag{6}$$

where $\bar{u}$ and $\bar{v}$ are the velocity components in the directions of x and y respectively T,C are temperature, Concentration respectively ρ is the density of the fluid μ is the dynamic viscosity of the fluid is the permeability of the porous medium σ is the electrical conductivity μ_e is the magnetic permeability, Ho is the strength of the magnetic field, D_B is the molecular diffusivity $k_c^{'}$ is the coefficient of chemical reaction β_T are the coefficients of thermal expansion β_c is the volumetric coefficient of mass fraction, qr is the radiative heat flux g is the acceleration due to gravity , $k_c^{'}$ is the chemical reaction coefficient. $\sigma^{\bullet}$ is the Stefan-Boltzman constant , β_R mean absorption coefficient. A'_{11} and B'_{11} are the coefficient of space and temperature dependent heat source/sink respectively

The fluid properties are assumed to be constant except fluid viscosity and thermal conductivity. Therefore we assume that the viscosity of the fluid is to be an inverse function of the temperature and it can be expressed as[Lai and Kulacki [7]]

$$\frac{1}{\mu} = \frac{1}{\mu_\infty}(1 + \delta(T - T_\infty) \ or \ \frac{1}{\mu} = b((T - T_\infty) \tag{7}$$

where $b = \frac{\delta}{\mu_\infty}$ and $T = T_\infty - \frac{1}{\delta}$.

Both b and Tr are constants and their values depend on the reference state and the thermal property of the fluid i.e δ .In general b>0 for liquids and b<0 for gases θ_r which is defined by

$$\theta_r = \frac{T_r - T_\infty}{T_w - T_\infty} = -\frac{1}{\delta(T_w - T_\infty)} \tag{8}$$

is constant.

The parameter θ_r was first introduced by Ling and Dybbs. It is important to note that for $\delta \to 0 (i.e\ \mu = \mu_\infty = constant)$ then $\theta_r \to \infty$ the effect of viscosity is negligible. The value of θ_r is determined by the temperature difference $(T_w - T_\infty)$ and viscosity δ of the fluid in consideration. A smaller values of θ_r is implies either the fluid viscosity changes considerably or the temperature difference is high. On the other hand for a larger values of θ_r implies either $(T_w - T_\infty)$ or δ is small and therefore the effects of variable viscosity can be neglected. In either case the influence of variable viscosity plays very important role and the liquid viscosity varies differently with temperature than that of gases. Therefore θ_r is positive for gases and negative for liquids respectively.

Also we assume that the fluid thermal conductivity α is to be varying as a linear function of temperature in the form [Seddeek and Salem[12]]

$$\alpha = \alpha_o(1 + E(T - T_\infty))$$

Where α_o is the thermal diffusivity at the wavy surface temperature Tw and E is a constant depending on the nature of the fluid. It is worth mentioning here that E is positive for fluids such as air and E is negative for fluids such as lubrication oils. This can be written in the non-dimensional form [Slattery[15]] as

$$\alpha = \alpha_o(1 + \beta\theta) \tag{9}$$

Where $\beta = E(T - T_\infty)$ is the thermal conductivity parameter. The variation of β can be taken in the range $-0.1 \le \beta \le 0$ for lubrication oils $0 \le \beta \le 0.12$ for water and $0 \le \beta \le 6$ for air.

In view of the continuity equation(2) we define the stream function ψ as

$$\bar{u} = \frac{\partial \bar{\psi}}{\partial \bar{y}}, \quad v = -\frac{\partial \bar{\psi}}{\partial \bar{x}} \tag{10}$$

Introducing non-dimensional variables as

$$x = \frac{\bar{x}}{l}, y = \frac{\bar{y}}{l}, a = \frac{\bar{a}}{l}, \sigma = \frac{\bar{\sigma}}{l}, \psi^* = \frac{\bar{\psi}}{l}, \theta = \frac{T - T_\infty}{T_w - T_\infty}, \phi = \frac{C - C\infty}{C_w - C_\infty} \tag{11}$$

the equations (3)-(5) reduce to

$$\left(\frac{1}{\theta - \theta_r}\right)\left(\frac{\partial\theta}{\partial y}\frac{\partial\psi^\bullet}{\partial y} - \frac{\partial\theta}{\partial x}\frac{\partial\psi^\bullet}{\partial x}\right) + \left(\frac{\partial^2\psi^\bullet}{\partial x^2} + \frac{\partial^2\psi^\bullet}{\partial y^2}\right) + + Ra\left(1 - \frac{\theta}{\theta_r}\right)\left(\frac{\partial\theta}{\partial y} + N_r\frac{\partial C}{\partial y}\right) - M^2\frac{\partial^2\psi^\bullet}{\partial y^2} \tag{12}$$

$$\left(\frac{\partial\theta}{\partial x}\frac{\partial\psi^\bullet}{\partial y} - \frac{\partial\theta}{\partial y}\frac{\partial\psi^\bullet}{\partial x}\right) = \left(1 + \beta\phi + \frac{4Rd}{3}\right)\left(\frac{\partial^2\theta}{\partial x^2} + \frac{\partial^2\theta}{\partial y^2}\right) + \beta\left(\left(\frac{\partial\theta}{\partial x}\right)^2 + \left(\frac{\partial\theta}{\partial y}\right)^2\right) + + \left(A11\frac{\partial\psi^\bullet}{\partial y} + B11\theta\right) \tag{13}$$

$$Le\left(\frac{\partial\phi}{\partial x}\frac{\partial\psi^\bullet}{\partial y} - \frac{\partial\phi}{\partial y}\frac{\partial\psi^\bullet}{\partial x}\right) = Le\left(\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2}\right) - Le\gamma(1 + \delta\theta)Exp\left(-\frac{E1}{(1 + \delta\theta)}\right)\phi \tag{14}$$

Where $Ra = \frac{\beta_T g(T_w - T_\infty)l}{\alpha_o \nu}$ is the Darcy-Rayleigh number $\nu = \frac{\mu_\infty}{\rho}$ is the kinematic viscosity of the fluid,

$Rd = \frac{4\sigma^* T_\infty^3}{k_f \beta_R}$ is the Radiation parameter,

$M^2 = \frac{\sigma\mu_e^2 H_0^2 l^2}{\mu}$ .is the magnetic parameter.

$Le = \frac{\nu}{D_B}$ is the Lewis number $\gamma = \frac{kcl^2}{D_B}$ is the chemical reaction parameter , $k_c^{'} = k_c x^{-1}$,

$N = \frac{\beta_c(C_w - C_\infty)}{\beta_0(T_w - T_\infty)}$ is the buoyancy ratio , $\theta_w = \frac{T_w}{T_\infty}$

, $E1 = \frac{E_n}{kT}$ is the activation energy parameter,

$\delta = (\theta_w - 1)$ is the temperature difference parameter.

The transformed boundary conditions are

$$\psi^\bullet = 0, \theta = 1, \phi = 1 \ \ at \ \ y = aSin(x)$$

$$\frac{\partial\psi^\bullet}{\partial y} \to 0, \theta \to 0, \phi \to \infty \ \ as \ \ y \to \infty \tag{19}$$

We can transform the effect of wavy surface from the boundary conditions into the governing equations by using suitable coordinate transformation with boundary layer scaling for the case of free convection .The Cartesian coordinates(x,y) are transformed into the new variables(ξ, η).

We incorporate the effect of effect of wavy surface and the usual boundary layer scaling into the governing equations(16)-(18) for free convection using the transformations and $Ra \to \infty$ (i.e boundary layer approximation)

$$x = \xi, \hat{\eta} = \frac{y - aSin(x)}{\xi^{1/2} Ra^{-1/2}}, \psi^{\bullet} = Ra^{1/2}\psi$$

(20)

These transformations are similar to those presented in for instance Rees and Pop[11]. We obtain the following boundary layer equations :

$$\left(\frac{Be^{-B\theta}}{\theta-\theta_r}\right)(1+a^2Cos^2\xi)\frac{\partial\theta}{\partial\eta}\frac{\partial\psi}{\partial\eta} + e^{-B\theta}(1+a^2Cos^2\xi)\frac{\partial^2\psi}{\partial\eta^2} = Ra\,\xi^{1/2}\left(1-\frac{\theta}{\theta_r}\right)\left(\frac{\partial\theta}{\partial\eta} + N\frac{\partial\phi}{\partial\eta}\right) - M^2\frac{\partial^2\psi}{\partial\eta^2}$$

(21)

$$\xi^{1/2}\left(\frac{\partial\theta}{\partial\xi}\frac{\partial\psi^{\bullet}}{\partial\eta} - \frac{\partial\theta}{\partial\eta}\frac{\partial\psi^{\bullet}}{\partial\xi}\right) = (1+a^1Cos^2(\xi))\left((1+\beta\phi+\frac{4Rd}{3})\left(\frac{\partial^2\theta}{\partial\eta^2}\right) + \beta\left(\frac{\partial\theta}{\partial\eta}\right)^2\right) + \left(A11\frac{\partial\psi^{\bullet}}{\partial\eta} + B11\theta\right)$$

(22)

$$\xi^{1/2}Le\left(\frac{\partial\phi}{\partial\xi}\frac{\partial\psi}{\partial\eta} - \frac{\partial\phi}{\partial\eta}\frac{\partial\psi}{\partial\xi}\right) = (1+a^2Cos^2(\xi))\left(\frac{\partial^2\phi}{\partial\eta^2}\right) - Le\gamma(1+\delta\theta)\phi Exp\left(-\frac{E1}{1+\delta\theta}\right) + (1+a^2Cos^2(\xi))$$

(23)

III. SOLUTION METHODOLOGY

We now introduce the following similarity variables as

$$\eta = \frac{\bar{\eta}}{(1+a^2Cos^2(\xi))}, \psi = \xi^{1/2} f(\eta), \theta = \theta(\eta). \phi = \phi(\eta)$$

In equations(20)-(23)we obtain a system of ordinary differential equations as follows:

$$e^{-B\theta} f'' + \left(\frac{Be^{-B\theta}}{\theta-\theta_r}\right)\theta' f' - \frac{M^2}{(1+a^2Cos^2\xi)} f'' = Ra\left(1-\frac{\theta}{\theta_r}\right)(\theta' + N_r\phi')$$

(24)

$$\beta(\theta')^2 + \left(1+\beta\theta+\frac{4Rd}{3}\right)\theta'' + \frac{1}{2} f\theta' + (A11f' + B11\theta)$$

(25)

$$\phi'' + \frac{Le}{2} f\phi' - LeRa^{-1}\gamma(1+\delta\theta)(1+a^2Cos^2(\xi))\phi Exp\left(-\frac{E1}{1+\delta\theta}\right) = 0$$

(26)

where prime denotes differentiation with respect to η. The corresponding boundary conditions are

$f = 0$ θ=1 ϕ =1 at $\eta = 0$

(27)

$f' \to 0$ θ→0 ϕ →0 as $\eta \to \infty$

In equation(23) the radiation parameter $Rd = \frac{4\sigma^* T_\infty^3}{k_f \beta_R}$ means that the rate of thermal radiation contribution relative to the thermal conditions. As $Rd \to \infty$ influence of thermal radiation is high in the boundary layer regime. For $Rd \to 0$ the term 4Rd/3 tends to zero. For Rd=1thermal radiation and thermal conduction will give equal contribution.

The main results of practical interest in many applications are Skin friction coefficient , heat transfer coefficient ,mass transfer coefficient at the surface.

The Skin friction coefficient (Cf) is given by

$$C_f = \frac{f''(0)(1+a^2Cos^2(\xi))Ra^{1/2}}{(1`+M^2 + a^2Cos^2(\xi))}$$

The heat and mass transfer coefficients are expressed in terms of Nusselt and Sherwood numbers Nux Shx. Nusselt number Nux and Sherwood number Shx are given by

$$Nux = \frac{xq_w}{\alpha_o(T_w - T_\infty)} \quad Shx = \frac{xm_w}{D_b(C_w - C_\infty)}$$

(28)

where q_w is the heat flux on the wavy surface and is defined by

$$q_w = -\alpha_0 \bar{n}.\nabla T \text{ and } \bar{n} = \left(-\frac{aCos(\xi)}{\sqrt{(1+a^2Cos^2(\xi))}}, \frac{1}{\sqrt{(1+a^2Cos^2(\xi))}}\right)$$

is the unit normal vector to the wavy surface α_0 is the effective porous medium thermal conductivity.

Therefore

$$Nu_\xi = -\frac{\theta'(0)Ra_\xi^{1/2}}{\sqrt{(1+a^2Cos^2(\xi))}}$$

$$Sh_\xi = -\frac{\phi'(0)Ra_\xi^{1/2}}{\sqrt{1+a^2Cos^2(\xi)}}$$

(29)

IV. CONPARISON

In the absence of chemical reaction ($\gamma = 0$), heat sources (A11,B11=0), activation energy (E1=0,δ-0) and magnetic field(M=0) the results are in good agreement with **Mallikarjuna** [9]

Table.1

$NuxRa^{-1/2}$		**$ShxRa^{-1/2}$**	**$NuxRa^{-1/2}$**		**$ShxRa^{-1/2}$**	
θr	β	a	***Mallikarjun1***[9]		**Present Results**	
-2	0.5	0.5	0.3056	0.4433	**0.3059**	**0.4434**
-4	0.5	0.5	0.2976	0.4301	**0.2980**	**0.4306**
-10	0.5	0.5	0.2952	0.4264	**0.2959**	**0.4250**
-2	0.1	0.5	0.3702	0.4034	**0.3703**	**0.4038**
-2	0.3	0.5	0.3232	0.4163	**0.3235**	**0.4167**
-2	0.5	0.1	0.6525	0.2755	**0.6528**	**0.2759**
-2	0.5	0.3	0.5543	0.2555	**0.5547**	**0.2562**
-2	0.5	0.5	0.3913	0.3899	**0.3914**	**0.3900**
-2	0.5	0.5	0.3555	0.3999	**0.3556**	**0.3996**

V. RESULTS AND DISCUSSION

The aim of this analysis is to investigate the combined influence of variable viscosity, activation energy, thermal conductivity on the flow characteristics**.**

The variation of the non-dimensional velocity, temperature and concentration profiles with η for different values of temperature dependent viscosity parameter(θr), thermal conductivity parameter(β), radiation parameter (Rd), activation energy parameters(E1,δ),heat source parameter(A11,B11), Buoyancy parameter (N), magnetic parameter(M), Rayleigh number(Ra), Lewis parameter(Le), surface amplitude of the wavy surface(*a) and* stream wise coordinate (ξ) are presented in figs.2a-2c to 10a-10c.

Figs.2a-2c illustrate the variation of velocity, temperature and concentration with Rayleigh number(Ra) and magnetic parameter(M).It can observed from the profiles that the axial velocity enhances in the flow region. An increase in Ra reduces the temperature and concentration .This may be attributed to the fact that the thickness of the thermal and solutal boundary layers reduce with increasing values of Ra. The velocity reduces, temperature and concentration enhance with increasing values of magnetic parameter. This is due to the fact that in increase in M grows the thickness of the thermal and solutal boundary layers(figs.2a-2c)

Fig.3a-3c depict the variation of velocity temperature and concentration with magnetic parameter (M). From the profiles we find that the velocity temperature and concentration reduce with increasing values of magnetic parameter. This is due to the fact that in increase in M reduces the thickness of the thermal and solutal boundary layer.

Figs.4a-4c depict the variation of velocity temperature and concentration with buoyancy ratio(N) thermal conductivity(β). It can be seen from the profiles that when the molecular buoyancy force dominates over the thermal buoyancy force the velocity enhances in the region when the buoyancy forces are in the same direction in the region(fig.3a). The temperature and concentration reduce with increase in N>0 in the flow region (figs.3b&c) . Fig.3a shows the variation of velocity with β.In this case the velocity is found to depreciate in the flow region (0,4).From fig.3b &3c we find that as the thermal conductivity parameter β increases the temperature increases and the concentration decreases. This is due to the thickening of the thermal and thinning of the solutal boundary layers as a result of increasing values of thermal conductivity

. Figs.4a-4c represent u, θ and ϕ with space dependent heat source parameter(A11).From fig.4a we find that the an increase in the strength of the space dependent heat source(A11>0)increase the velocity, temperature and concentration, while they experience a depreciation with heat sink(A11<0)(fig. 4a-4c).

Figs.5a-5c represent u, θ and ϕ with temperature dependent heat source parameter(B11).From fig.5a we find that the an increase in the strength of the heat generating source(B11>0) leads to an enhancement in the velocity, temperature and concentration while they experience a depreciation with heat absorbing source(B11<0)(fig. 5a-5c).This may be attributed to the fact that in the presence of heart generating source, heat is generated and in the case of absorbing source, heat is absorbed in the boundary layer region.

The variation of non-dimensional velocity, temperature and concentration profiles with η for different values of temperature dependent viscosity parameter (θr)and radiation parameter(Rd) is illustrated in figs.6a-6c.It is found that from fig.6a that the velocity of the fluid reduces in the flow region. This can be explained physically as the parameter θr increases there is decay in the boundary layer thickness. From fig.6b&6c we notice that the temperature increases and the concentration profiles decreases with increasing values of θr. This can be attributed to the fact the an increase in θr grows the thickness of the thermal and decays the thickness of the solutal boundary layer which results in an increment in temperature and enhancement in the concentration in the flow region. An increase in radiation parameter(Rd) reduces the velocity in the flow region (0,4.0).This means that the thickness of the momentum boundary layer decays in the flow region. Higher values of Rd leads to a growth in thickness of the thermal and solutal boundary layers which results in n enhancement in temperature and concentration in the flow region(figs.6a-6c)..

Figs.7a-7c represent the velocity, temperature and concentration with chemical reaction parameter(γ).It can be seen from the profiles that the velocity reduces in the degenerating chemical reaction case and grows in the generating case in the region. An increase in γ>0 enhances the temperature and reduces the concentration in the flow region,w hile with γ<0,temperature and concentration enhance in the flow region(figs.7b&7c).

Figs.11a-11c represent the effect of dissipation (Ec) on the velocity, temperature and concentration. It can be seen from the profiles that higher the dissipation larger the velocity in the region(0,1.0) and smaller in the remaining flow region .From figs.11b&11c we find that the temperature increases and the concentration reduces with higher values of Ec in the flow region .

The effect of activation energy (E1) and temperature difference parameter(δ) on u,θ and ϕ can be seen from figs.8a-8c.The velocity, concentration elevate, temperature depreciates with rising values of activation energy parameter(E1).An increase in temperature difference parameter(δ) decays the momentum and thermal boundary layer thickness and grows the solutal layer thickness. The activation energy (E1) concentrates more as a result of function of Arrhenius. Generally, Activation energy is the minimum amount of energy that is required for a chemical reaction to stimulate atoms or molecules in the reaction. There should be a considerable number of atoms whose Activation energy is less than or equal to translational energy in a chemical reaction. In many engineering applications, activation energy may be considered as a better coolant.

Figs.9a-9c represent u,θ and ϕ with surface amplitude of the wavy '*a*' and stream-wise coordinate(ξ). From fig.9a&b,we notice that the velocity and temperature reduces with amplitude 'a' and enhance with stream-wise coordinate (ξ) in the entire flow region. The concentration(ϕ) enhances with 'a' and reduces w3ith 'ξ' in the flow region (fig.9c).

An increase in Lewis number (Le) decreases the velocity and concentration, enhances the temperature in the region(0,4.0).The velocity, temperature and concentration experience a depreciation with increase in index number(n)(figs.10a-10c)

The variation of skin friction for different parametric variations is presented in table.2. From the tabular values we find that the magnitude of the skin friction enhances with increase in Ra and buoyancy ratio(N).$|C_f|$ increases at the wall with increase in thermal conductivity(β) and radiation parameter(Rd). Higher the viscosity parameter(θr) or higher activation energy(E1)smaller the skin friction at the wall. Higher the strength of the space /temperature dependent heat source (A11,B11) larger the skin friction at η=0 ansds smaller with A11<0,B11<0. Cf enhances at the wall in the degenerating/generating chemical reaction cases at the wall. Higher the Lewis number(Le)or temperature difference parameter(δ) or index number(n)larger Cf at η=0.Cf reduces with higher values of activation parameter(E1)/ amplitude (a' of the surface/stream-wise coordinate(ξ)at the wall.

The rate of heat transfer(Nu) and mass transfer(Sh) at the wall is displayed in table.2. From the tabular values we find that rate of heat transfer at the wall decreases ,rate of mass transfer increase with inAn increase in magnetic field(M)/viscosity parameter(θr)/temperature difference parameter(δ) enhances the Nusselt number and decays the Sherwood number at η=0.Higher the values of thermal conductivity parameter(β)/radiation parameter(Rd)/ amplitude of surface(a) smaller the Nu and Sh at η=0.Nu and Sh at the wall grows with stream-wise coordinate(ξ)/index number(n)/Lewis number(Le).The rate of heat and mass transfer grows with A11>0,B11>,γ>0 and decay with higher values of A11<0,B11<0,γ<0 at η=0.

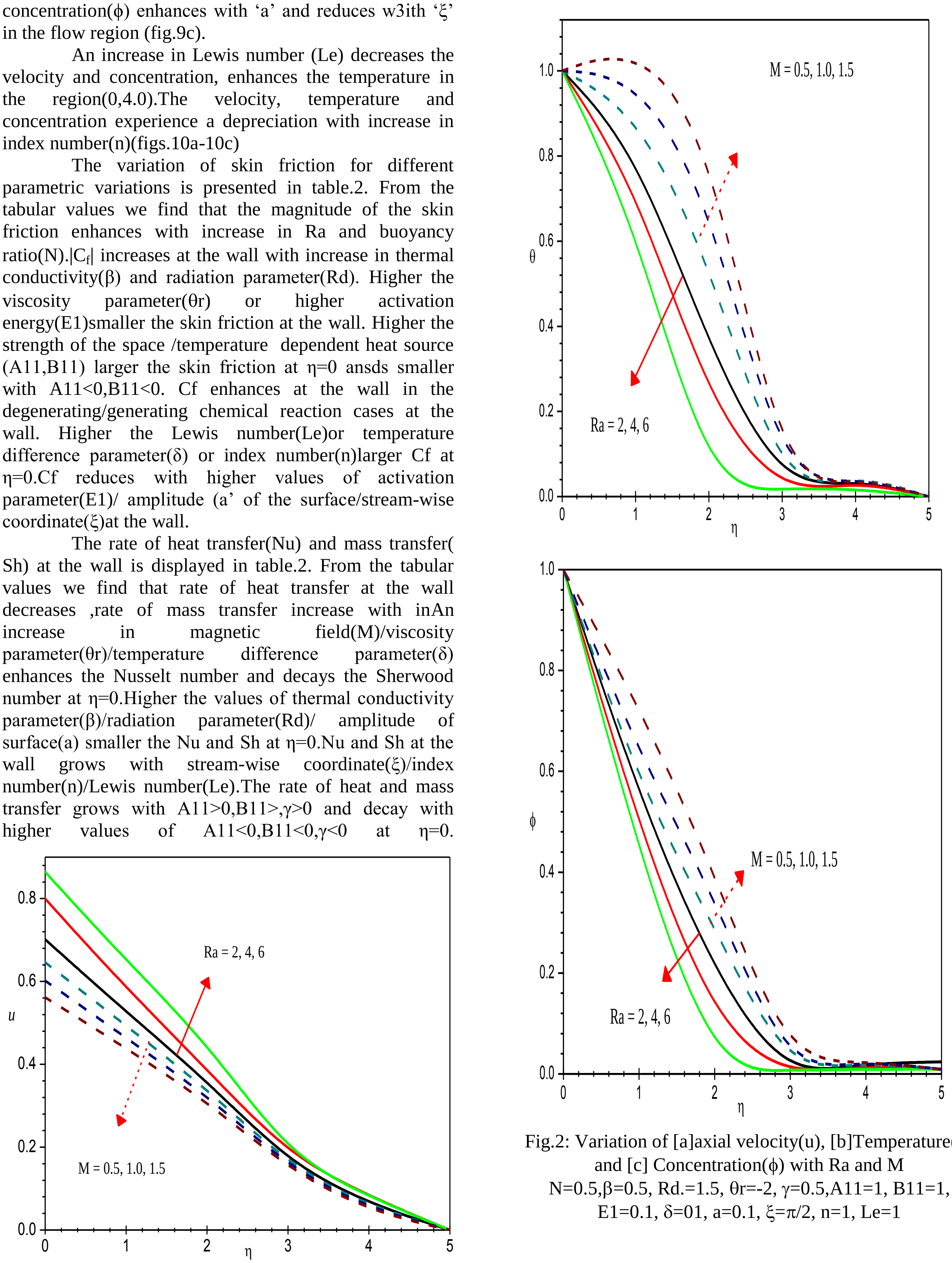

Fig.2: Variation of [a]axial velocity(u), [b]Temperature(θ) and [c] Concentration(ϕ) with Ra and M N=0.5,β=0.5, Rd.=1.5, θr=-2, γ=0.5,A11=1, B11=1, E1=0.1, δ=01, a=0.1, ξ=π/2, n=1, Le=1

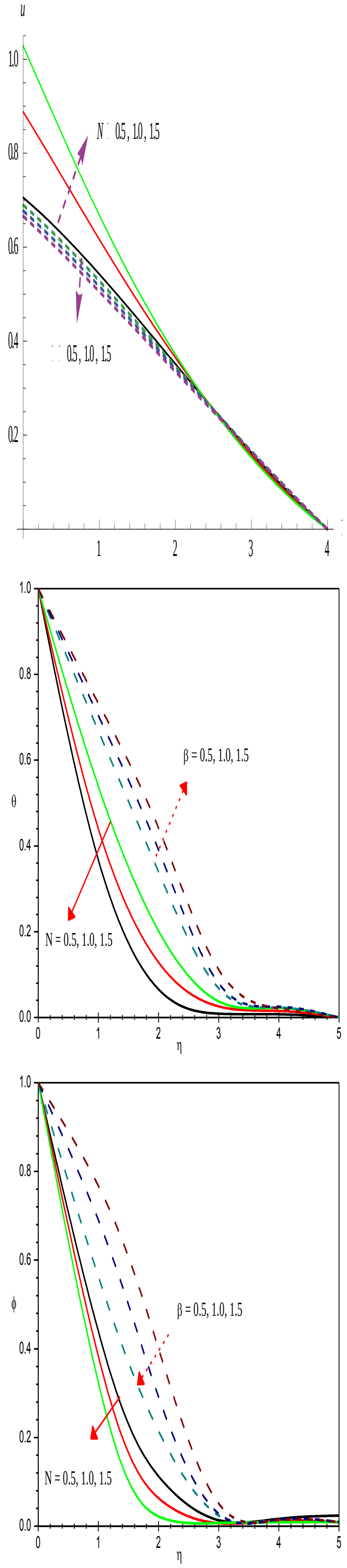

Fig.3: Variation of [a] axial velocity(u), [b]Temperature(θ) and [c] Concentration(ϕ) with β and N
Ra=2, M=0.5, Rd.=1.5, θr=-2, γ=0.5,A11=1, B11=1, E1=0.1, δ=01, a=0.1, ξ=π/2, n=1, Le=1

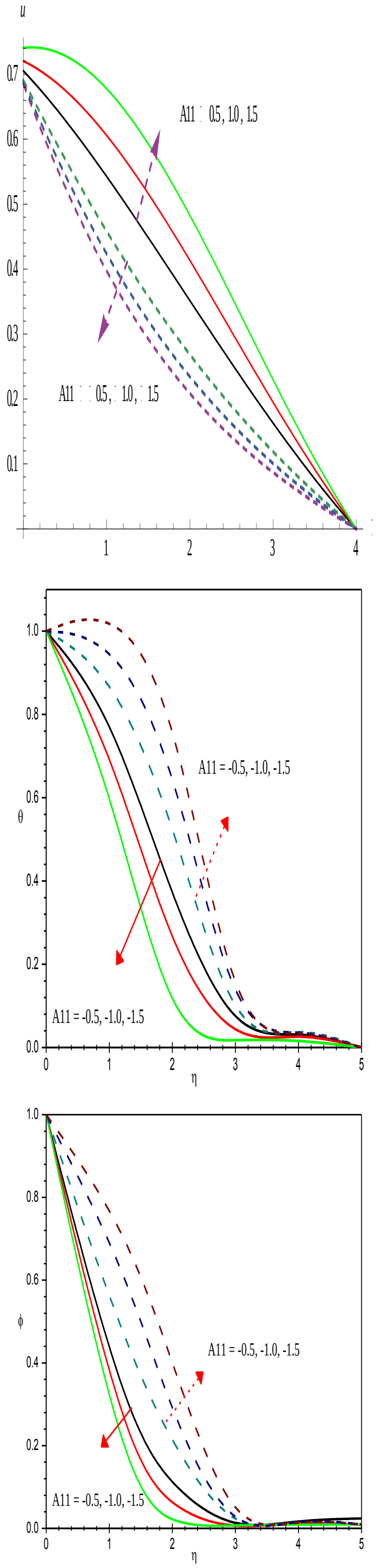

Fig.4: Variation of [a]axial velocity(u), [b]Temperature(θ) and [c] Concentration(ϕ) with A11
Ra=2, M=0.5, N=0.5,β=0.5, Rd.=1.5, θr=-2, γ=0.5, B11=1, E1=0.1, δ=01, a=0.1, ξ=π/2, n=1, Le=1

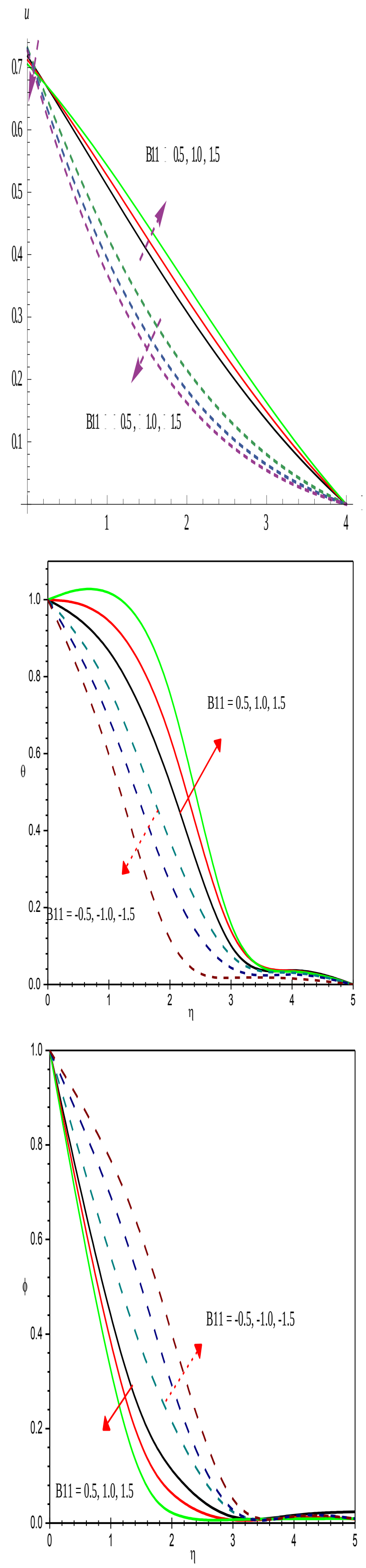

Fig.5: Variation of [a]axial velocity(u), [b]Temperature(θ) and [c] Concentration(ϕ) with B11
Ra=2, M=0.5, N=0.5,β=0.5, Rd.=1.5, θr=-2, γ=0.5,A11=1, E1=0.1, δ=01, a=0.1, ξ=π/2, n=1, Le=1

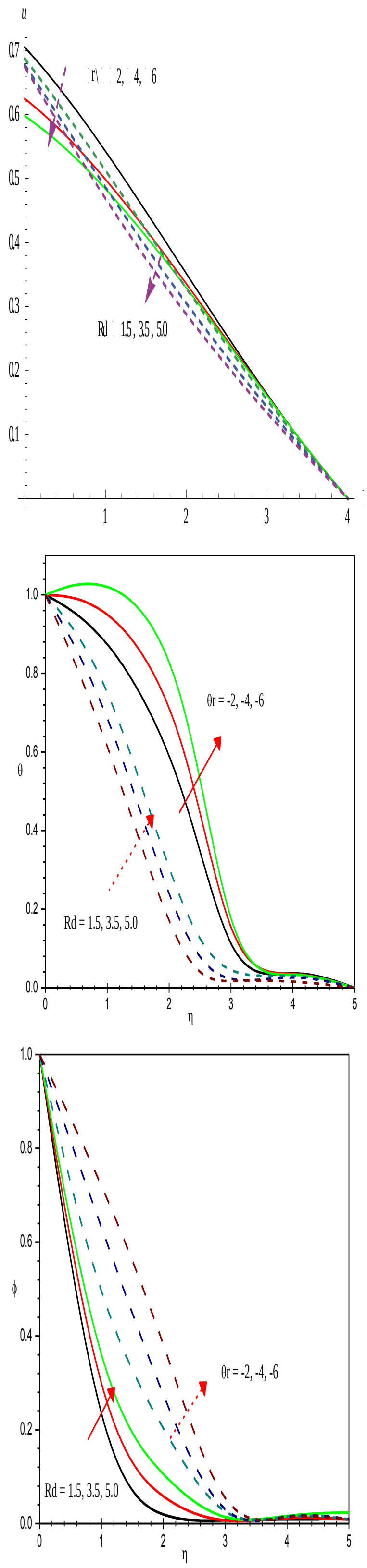

Fig.6: Variation of [a]axial velocity(u), [b]Temperature(θ) and [c] Concentration(ϕ) with θr and Rd
Ra=2, M=0.5, N=0.5,β=0.5, γ=0.5,A11=1, B11=1, E1=0.1, δ=01, a=0.1, ξ=π/2, n=1, Le=1

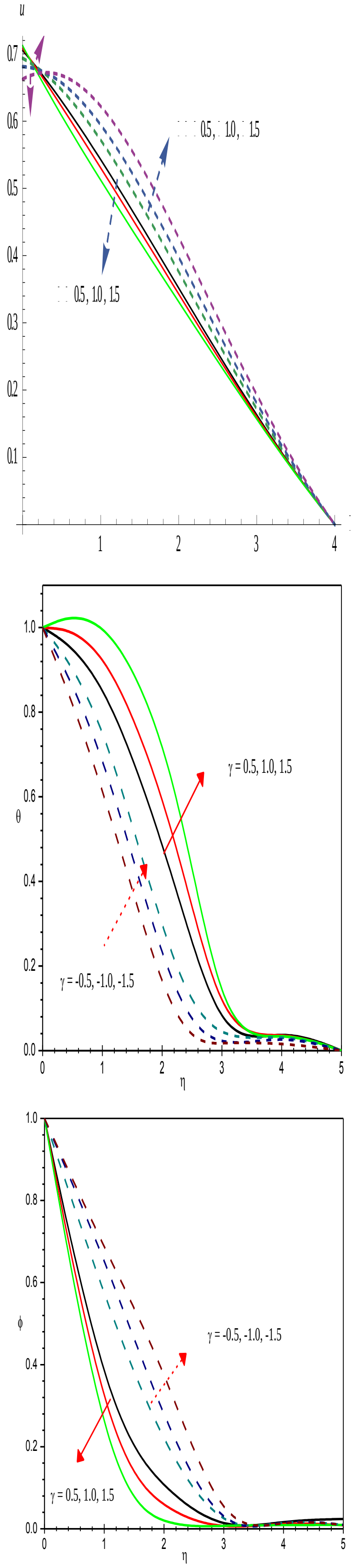

Fig.7: Variation of [a]axial velocity(u), [b]Temperature(θ) and [c] Concentration(ϕ) with γ
Ra=2, M=0.5, N=0.5,β=0.5, Rd.=1.5, θr=-2, A11=1, B11=1, E1=0.1, δ=01, a=0.1, ξ=π/2, n=1, Le=1

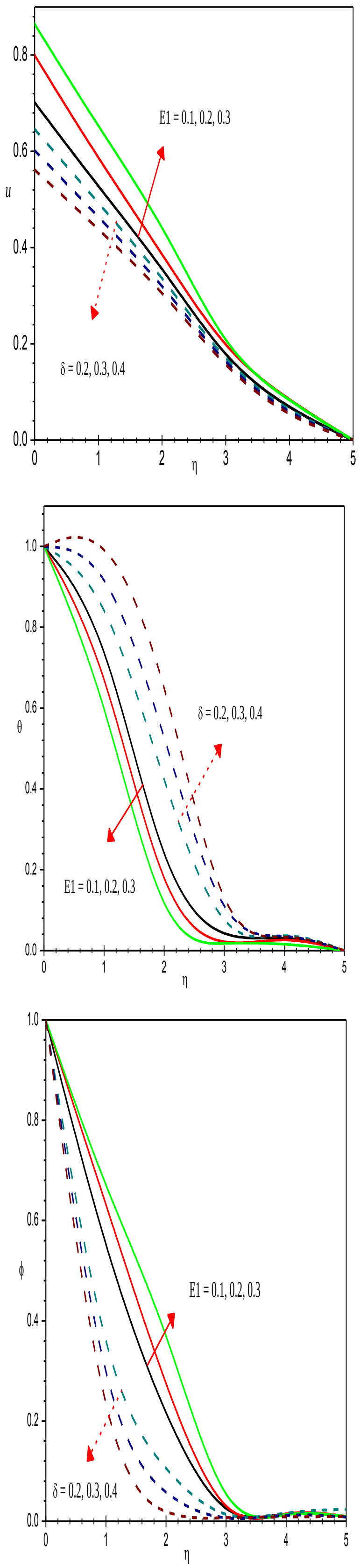

Fig.8: Variation of [a]axial velocity(u), [b]Temperature(θ) and [c] Concentration(ϕ) with E1 and δ
Ra=2, M=0.5, N=0.5,β=0.5, Rd.=1.5, θr=-2, γ=0.5,A11=1, B11=1, a=0.1, ξ=π/2, n=1, Le=1

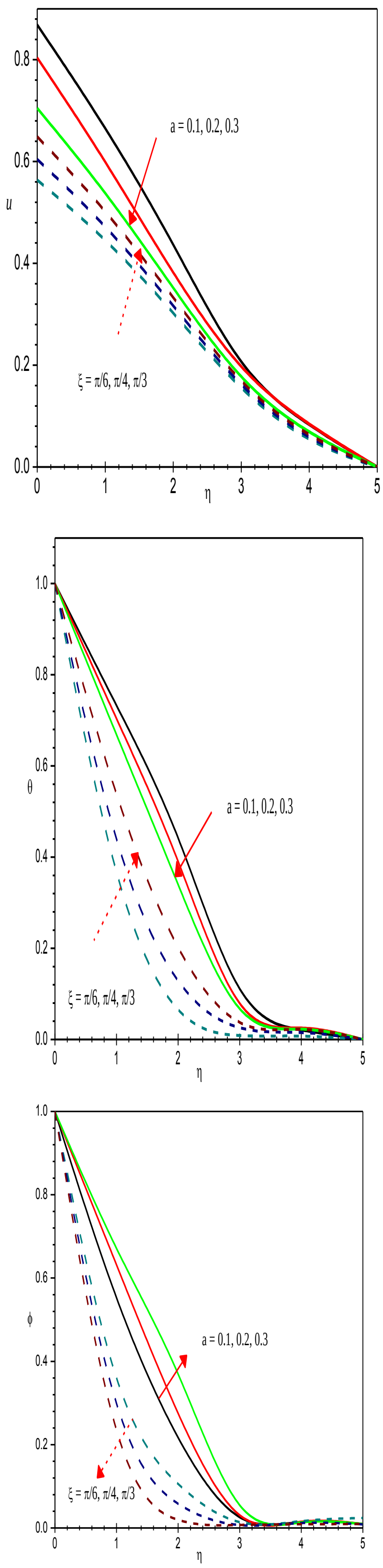

Fig.9: Variation of [a]axial velocity(u), [b]Temperature(θ) and [c] Concentration(ϕ) with a and ξ
Ra=2, M=0.5, N=0.5,β=0.5, Rd.=1.5, θr=-2, γ=0.5,A11=1, B11=1, E1=0.1, δ=01, n=1, Le=1

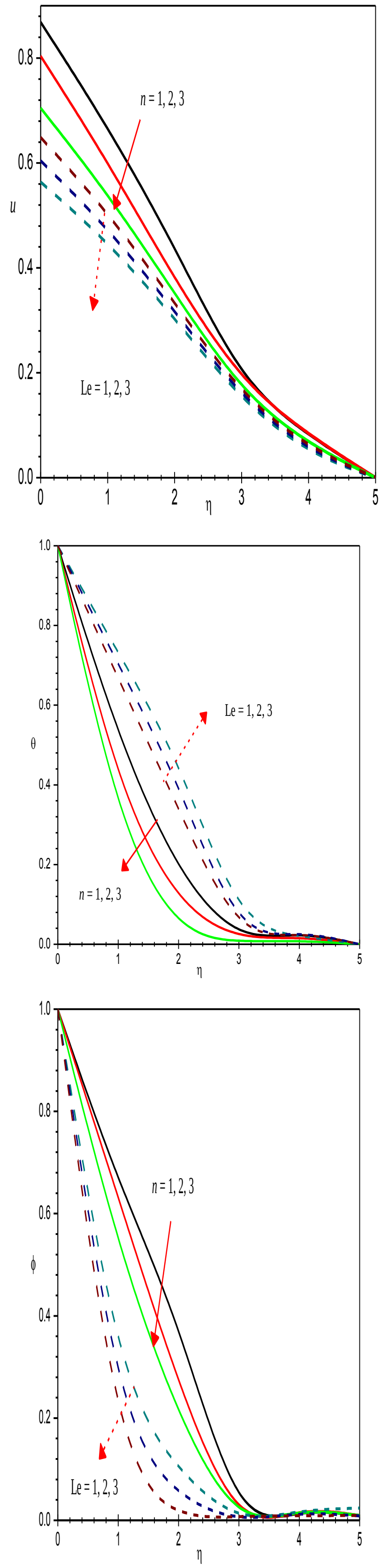

Fig.10: Variation of [a]axial velocity(u), [b]Temperature(θ) and [c] Concentration(ϕ) with *n* and *Le*
Ra=2, M=0.5, N=0.5,β=0.5, Rd.=1.5, θr=-2, γ=0.5,A11=1, B11=1, E1=0.1, δ=01, a=0.1, ξ=π/2

Table - 2

Skin Friction (τ), Nusslet number (Nu) and Sherwood Number (Sh) at $\eta = 0$

Parameter		τ(0)	Nu(0)	Sh(0)	Parameter		τ(0)	Nu(0)	Sh(0)
Ra	2	-0.136462	-0.101377	0.520884	a	0.1	-0.146462	-0.103377	0.520884
	4	-0.297620	-0.030141	0.603800		0.2	-0.133069	-0.092983	0.520523
	6	-0.496100	0.024086	0.677067		0.3	-0.146792	-0.080881	0.519996
M	0.5	-0.116534	-0.112692	0.508750	ξ	π/6	-0.137093	-0.100122	0.520833
	1.0	-0.101339	-0.121837	0.498938		π/4	-0.136462	-0.101377	0.520884
	1.5	-0.088421	-0.130022	0.490120		π/3	-0.135826	-0.102643	0.520939
N	0.5	-0.136462	-0.101377	0.520884	n	1	-0.136462	-0.101377	0.520884
	1.0	-0.262369	-0.077923	0.551339		2	-0.141541	-0.101836	0.535641
	1.5	-0.366961	-0.060819	0.573785		3	-0.146528	-0.102285	0.550129
β	0.5	-0.141305	-0.0877593	0.516736	Le	1	-0.177868	-0.106094	0.642021
	1.0	-0.144633	-0.0781283	0.513368		2	-0.212896	-0.110261	0.745113
	1.5	-0.147703	-0.0692741	0.510124		3	-0.243582	-0.113795	0.835461
Rd	1.5	-0.160457	-0.0466355	0.512661	A11	0.5	-0.136462	-0.101377	0.520884
	3.5	-0.188421	0.0939148	0.505674		1.0	-0.162572	-0.264107	0.536868
	5.0	-0.211348	0.0624281	0.501033		1.5	-0.247755	-0.43954	0.554486
θ_r	-2	-0.136462	-0.101377	0.520884		-0.5	-0.2042139	0.137177	0.498867
	-4	-0.098425	-0.114513	0.505203		-1.0	-0.1867546	0.246354	0.489585
	-6	-0.085629	-0.119016	0.499875		-1.5	-0.1456788	0.335876	0.482404
γ	0.5	-0.105916	-0.115702	0.504643	B11	0.5	-0.136443	0.0249835	0.513315
	1.5	-0.158657	-0.120686	0.679168		1.0	-0.196462	-0.0372963	0.517126
	-0.5	-0.061619	-0.103198	0.303517		1.5	-0.339763	-0.1013773	0.520884
	-1.5	0.0832165	-0.0960832	0.119804		-0.5	-0.128244	0.333783	0.492341
E1	0.1	-0.136462	-0.101377	0.520884		-1.0	-0.098765	0.467819	0.482657
	0.2	-0.131059	-0.100886	0.505204		-1.5	-0.067854	0.566257	0.475662
	0.3	-0.126851	-0.100502	0.492988					
δ	0.1	-0.142978	-0.101966	0.539816					
	0.2	-0.148786	-0.102488	0.556686					
	0.3	-0.154484	-0.102996	0.573234					

CONCLUSIONS

In this analysis we discuss the effect of activation energy, temperature difference parameter, viscosity parameter, thermal conductivity, irregular heat source, chemical reaction on the flow characteristics.The velocity, reduces the temperature and concentration with increasing values of Rayleigh number(Ra).This may due to the fact that the thickness of the thermal and solutal boundary layers decay with increasing values of Ra. Skin friction, Sherwood numbers enhance,Nusselt number decreases with increasing Ra.The velocity depreciates,temperature and concentration upsurges with higher values of Lorentz force.Cf,Sh decay and Nu grows at the wall with M.The velocity decays ,temperature and concentration increases in the flow region with higher values of θr.Cf,Sh decay,Nu grows with θr.An increase in the strength of the space / temperature dependent heat source upsurges velocity, temperature and concentration .Cf,Nu and Sh grows with A11>0,B11>0 and decays with A11<0,B11<0 at η=0.Increasing thermal conductivity parameter β increases temperature and reduces velocity,concentration .Cf,Sh grows,Nu decays with rising values of β.The velocity reduces, temperature and the concentration upsurges with Rd in the entire flow region. Cf grows,Nu and Sh decays with Rd.The velocity ,concentratrion reduces,temperature enhances in the degenerating chemical reaction case and in the generating case,u,θ,ϕ upsurge in the region.Cf,Nu,Sh grows with γ>0 and with γ<0,Nu,Sh decays at the wall.The velocity ,concentratioin enhance,temperature reduces with higher values of Activation energy(E1).Cf,Nu,Sh decay with E1 on the wall.Velocity,concentration reducer,temperaturer enhances with temperature difference parameter(δ).Cf,Nu grows,Sh decayus with δ.An increase in Lewis number (Le) increases temperature,reduces velocity,concentrtation in the flow region.Cf,Nu,Sh grow with Le at =0.An increase in amplitude 'a' reduces the velocity ,temperature and enhances Cf,Nu ,Sh decays with increasing values of 'a'.An increase in stream wise coordinate (ξ) increases the velocity ,temperature and decays concentration in the flow region.Cf decays,Nu,Sh grows with increasing values of 'ξ'.

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