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Computationally Efficient Optimal Power Flow Tracing

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Abstract— Power flow tracing is the method to decompose the line flows into multi commodity flows. Due to multiplicity in solution, Optimal Power Flow tracing has been proposed to choose the best solution in multiple solutions for a given objective. Min-max fair power flow tracing is well accepted cost and loss allocation. However, the size (number of variables and constraints) of min-max fair power flow tracing problem increases exponentially with the size of system. Therefore, in this paper, we propose a symbolic reduction step wherein variables which will be set to zero at optimal are identified a priori by a graph theoretic analysis. This leads to reduce number of variables in the LP problem. Further, the reduced problem can be solved efficiently by sparse LP method. The number of variables, constraints and the time required for solving a min-max fair power flow tracing problem has been reduced significantly with variable reduction technique. The variable reduction technique on IEEE 118 bus system has reduced 86%, 75% and 79% in number of variables, constraints and time respectively to solve min-max fair power flow tracing problem. A commercial optimization tool box which fails to solve the real life large scale system like NER grid (India) system without variable reduction. However, it is solved the same system in 21.83 seconds with variable reduction technique. Results on large 488 node Indian utility system illustrate the necessity of proposed approach.**Keywords— Real power flow tracing, Depth first search, Variable reduction, Linear Programming (LP), Transmission system usage cost and loss allocation, Object oriented design**

I. INTRODUCTION

Real power tracing is required to know the following:

- Generation Allocation
- Load allocation
- Flow Allocation

Above three contributions will be used to find the usage allocation. Usage allocation is one of the major issues experienced by the Electric Supply Industries. The usage allocation includes loss allocation associated to each path, cost assignment to transmission line pricing, congestion management, ancillary services and decision on scheduling of generators.

To compute the usage cost of a transmission, following methods have been proposed in literature [1].

1) **Postage stamp method:** Postage-stamp method, allocates the transmission system losses to the generators and loads proportional to their active generation or load consumption.
2) **Contract path methodology:** Contract path method selects a specific continuous path between the seller and buyer for wheeling transaction. All changes in the power flows through the transmission facilities which are not along the contract path are ignored. The path chosen must have "sufficient unused capacity" to carry the amount of power to be transported. The selection of the contract is, however, not usually based on the power ow study to identify the facilities actually involved in the transaction. This method is simple and provides a distinct way of settling financial liabilities to influenced parties along the contract path.
3) **MW-Mile method:** MW-Mile methodology may be regarded as the first pricing strategy proposed for the recovery of fixed transmission costs based on the actual use of transmission network. In this method, charges for each wheeling transaction are based on the measure of transmission capacity utilized in this transaction. However, it is difficult to segregate flow in a line in components of various simultaneous transactions.
4) **Proportionate sharing:** This method was proposed by J. Bialek [2]. The procedure assumes that the power reaching a certain node in the electric network is proportionately shared by all the paths going out from that node.

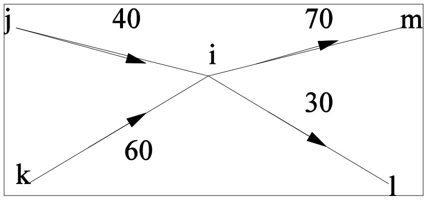

Fig. 1. Proportional sharing principle.

The principle of ``proportional sharing'' illustrated in Fig. 1 where four lines are connected to node i, two with inflows and two with outflows. The total flow through the node is $40+60=100$ units of which 40% is supplied by line $j-i$, and 60% by line $k-i$. As electricity is indistinguishable and each of the outflows down the line from node i depends only on the voltage gradient and the line impedance, it was assumed that each unit leaving the node contains the same proportion of the inflows as the total nodal flow. Hence the 70 units outflowing in line $i-m$ consists of $\frac{40\times70}{100}=28$ supplied by line $j-i$ and $\frac{60\times70}{100}=42$ supplied by line $k-i$.

Similarly, the 30 units outflowing in line $i-l$ consists of $\frac{40\times30}{100}=12$ supplied by line $j-i$ and $\frac{60\times30}{100}=18$ supplied by line $k-i$. Even though the *proportional sharing* assumption cannot be proved or disproved, its application leads to simplicity.

5) **Min-max fairness method** [4], [5]: A Min-max fair power flow tracing has been proposed in [4]. The min-max power flow tracing further improved in [5] for application in large systems. A solution is said to be min-max fair, if any reduction in, say, percentage usage costs (MU/MW) for one entity leads to increase in share of another entity, which has either equal or higher percentage cost. In absence of traceability constraint min-max fairness and postage stamp method would lead to identical cost distribution. An advantage of min-max fairness model over modified postage stamp method is that it guarantee`s fairness for each individual user, as within traceable constrained space, no other solution exists by which a higher cost entity (MU/MW) can off load usage costs to a lower cost entity. Min-max fairness also meets an important fairness criterion known as `aggregate invariance' which implies that artificial splitting of load or generator at a bus will not alter the final solution.
While optimal tracing is superior to normal tracing as one can rigorously impose a fairness related objective on the space of traceable solutions, there are certain computational challenges to be overcome. In particular number of variables in optimal tracing problem is increased drastically. Thus, as the system size increases, the optimization solver becomes inefficient.

In this paper, we proposed a variable reduction technique. The rest of the paper is organized as follows: the optimal tracing methods i.e. Tracing compliant modified postage stamp method and Min-max fairness tracing method are explained in Section II. Symbolic reduction technique to improve computational efficiency of optimal tracing problem is explained in Section-III. Object oriented design in unified modelling language (UML) of tracing engine is discussed in Section IV. Section V presents the results and Section VI concludes the paper.

II. Optimization Approach to Real Power Tracing: An Applications to Transmission Fixed cost Allocation [3]

The optimal tracing problem can be compactly defined as follows:

$$\textbf{Problem } OPT(x,y): \min_{(x,y)\in S} f(x,y) \qquad (1)$$

The set S represents the set of all possible tracing solutions and a specific set of x and y vectors represents a solution to generation and load tracing problem. It is shown that set can be characterized by a set of linear equality and inequality constraints. In fact, set is both compact and convex. This leads

to a linear constrained optimization problem. It models relationship between the flow entities and associated network usage costs. Constraints in **OPT** problem can be grouped into the following specifications

- flow specification constraints for series branches, i.e., transmission lines and transformers;
- source and sink specification constraints pertaining to shunts, e.g., generators and loads;
- conservation of commodity flow constraints.

A. Flow Specification Constraints

Traditionally, two types of tracing problems, viz., generation tracing and load tracing, are discussed in the literature. Generation tracing traces generator flows to loads, while load tracing traces load flows to generators. We first discuss modeling of the flow specification constraints for generation tracing problem.

1. *Generation Tracing:* Let P_{lm} (MW) be the flow on a line lm. Flow is supplied by generators $G_1, G_2, \ldots G_{nG}$ with components $P_{lm}^{G_1}, P_{lm}^{G_2} \ldots P_{lm}^{G_{nG}}$

$$P_{lm} = P_{lm}^{G_1} + P_{lm}^{G_2} + \ldots + P_{lm}^{G_{g_{nG}}} \qquad (2)$$

The component of generator G_k on lm line can be expressed as fraction x_{lm}^k of the total injection by generator G_k, i.e., . Therefore

$$P_{lm}^{G_k} = x_{lm}^k . P_{G_k} \quad (3)$$

$$P_{lm} = \sum_{k=1}^{ng} x_{lm}^k . P_{G_k}, \forall \; set \; of \; lines \quad (4)$$

Since the branch flows are known and x are unknown, flow equations for generation allocation can be written as follow:

$$[A_{flowd}][x_{flow}] = [b_{flow_d}] \quad (5)$$

2. *Load Tracing:* The power flow P_{lm} on line lm can also be expressed as a summation of load components, i.e.,

$$P_{lm} = P_{lm}^{L_1} + P_{lm}^{L_2} + \ldots + P_{lm}^{L_{n_L}} \quad (6)$$

The component of load (P_{Li}) on line lm is expressed as a fraction y_{lm}^i of load P_{Li} as follows:

$$P_{lm}^{L_i} = y_{lm}^i . P_{L_i} \quad (7)$$

$$P_{lm} = \sum_{i=1}^{n_L} y_{lm}^i . P_{L_i}, \forall \; set \; of \; lines \quad (8)$$

In matrix form, the flow equations for load allocation can be written as follows:

$$[A_{flow_u}][y_{flow}] = [b_{flow_u}] \quad (9)$$

B. *Source and Sink Specification Constraints*

1. *Generation Tracing:* In a generation tracing problem, it is necessary to write sink (load) constraints. They express the contribution of generators in loads. For a load (P_{L_i}), the contribution of various generators is governed by the following constraint:

$$P_{L_i} = P_{L_i}^{G_1} + P_{L_i}^{G_2} + \ldots + P_{L_i}^{G_{n_G}} \quad (10)$$

$$P_{L_i} = \sum_{k=1}^{n_G} x_i^k . P_{G_k}, i = 1, \ldots, n_L \quad (11)$$

In the matrix form, the load equations for generation allocation can be written as follows:

$$[A_{inj_d}][x_{inj}] = [b_{inj_d}] \quad (12)$$

2. *Load Tracing:* In the load tracing problem, it is necessary to model the share of loads in a generator. Let

$$P_{G_k} = P_{G_k}^{L_1} + P_{G_k}^{L_2} + \ldots + P_{G_k}^{L_{n_L}} i.e., P_{G_k} = \sum_{i=1}^{n_L} y_k^i P_{L_i}. \quad (13)$$

In matrix form, the generator equations for load allocation can be written as follows:

$$[A_{inj_u}][y_{inj}] = [b_{inj_u}] \quad (14)$$

C. *Conservation of Commodity flow constraints*

The conservation of flow constraints can be neatly expressed by using arc or bus incidence matrix M of the underlying graph. In the matrix, rows correspond to nodes and columns to arcs. The entry $M(i, j)$ is set to 1 if arc is outgoing at node; it is -1 if the arc is incoming at node; else, it is set to zero. The shunt arcs have one node as ground, which is not modeled in M. The corresponding entry in M is either 1 or -1, depending upon whether the arc represents load or generation.

1) *Generation Tracing:* Let M_d represents sub-matrix formed by considering series branches and shunt loads

$$M_d = [M_{n_b}, M_L] \quad (15)$$

$$[M_d][x^k] = [e_{\bar{k}}] \qquad k = 1 \ldots n_G \quad (16)$$

Where x^k represents the set of x-variables for lines and loads associated with the k^{th} generator. $\bar{k}$ is the node at which k^{th} generator is connected, and $e_{\bar{k}}$ is the k^{th} column of identity matrix.

Instead of partitioning x variables by generator numbers, they can be partitioned by series branch flow (x_{flow}) and shunt flow x_{inj} variables. The set of the continuity equations can be rearranged and written in block matrix notations with and the variable partition as follows:

$$\begin{bmatrix} A_{cont_{flow_d}} & A_{cont_{inj_d}} \end{bmatrix} \begin{bmatrix} x_{flow} \\ x_{inj} \end{bmatrix} = [b_{cont_d}] \quad (17)$$

2) *Load Tracing:* The following similar arguments as in generation tracing, the continuity equations for load tracing are given as follows:

$$[M_u][y^i] = [e_{\bar{i}}] \quad i = 1 \ldots n_L \quad (18)$$

where $\bar{i}$ is the node at which the i^{th} load is connected.

$$\begin{bmatrix} A_{cont_{flow_u}} & A_{cont_{inj_u}} \end{bmatrix} \begin{bmatrix} y_{flow} \\ y_{inj} \end{bmatrix} = [b_{cont_u}] \quad (19)$$

D. *Lossy Flow network*

For a lossy flow network, the flow reduces along the arc from the sending end to the receiving end. Under such situations, it is necessary to model two flow equations per line, one for the sending end and one for the receiving end. An alternative approach has been developed to restrict the number of variables and equations in the optimization problem to the corresponding lossless formulation. For taking care of lossy flow network we need to change the M_d and M_u. Let o_i is origin and d_i is destination of an arc i.

$$M_d^{loss}(o_i, i) = 1 \quad (20)$$

$$M_d^{loss}(d_i, i) = -\frac{P_{lm}^r}{P_{lm}^s}, \forall lm \in set\ of\ lines \quad (21)$$

$$M_u^{loss}(o_i, i) = \frac{P_{lm}^s}{P_{lm}^r}, \forall lm \in set\ of\ lines \quad (22)$$

$$M_u^{loss}(d_i, i) = -1 \quad (23)$$

E. Modeling of Boundary Conditions

In a megawatt flow network, if the results are consistent, then this difference, i.e., power dispatched by generator k to load i minus power received from generator k by load i, should correspond to the loss incurred in the generator-k load-i interaction. Let us define this loss as $loss_i^k$, where

$$loss_i^k = y_k^i P_{L_i} - x_i^k P_{G_k} \quad (24)$$

The above equation should satisfy the following loss properties of a network:

$$y_k^i P_{L_i} - x_i^k P_{G_k} \geq 0, \forall\ k \in \{1,2,\ldots,n_G\} \quad (25)$$

$\forall\ k \in \{1,2,\ldots,n_L\}$

$$P_{loss} = \sum_{k=1}^{n_G}\sum_{i=1}^{n_L} y_k^i P_{L_i} - x_i^k P_{G_k} \quad (26)$$

Equation (25) models the constraint that loss is a non-negative number, while (26) models the requirement that total generator-load interaction losses should be equal to the power loss in the system.

F. Explicit formulation

1) Objective Function for Transmission Fixed Cost Allocation Using Modified Postage Stamp Method: The transmission system usage cost per MW paid by a load can be worked out as follows:

$$trprice_{Load_{pu}}^i = \frac{\sum_{\forall lm} y_{lm}^i P_{L_i} \bar{c}_{lm}\alpha}{P_{L_i}}$$

$$= \sum_{\forall\ lm} y_{lm}^i \bar{c}_{lm}\alpha \quad (27)$$

The transmission system usage cost per MW paid by a generator can be worked out as follows:

$$trprice_{Gen_{pu}}^k = \frac{\sum_{\forall lm} x_{lm}^k P_{G_k} \bar{c}_{lm}(1-\alpha)}{P_{G_k}}$$

$$= \sum_{\forall lm} x_{lm}^k \bar{c}_{lm}(1-\alpha) \quad (28)$$

where $\bar{c}_{lm}$ is the cost of line per MW, α is the P. U. share of the load in total transmission cost, $0 \leq \alpha \leq 1$. Further, we assumed without loss of generality that $TSC_{net} = \sum_{\forall lines} \bar{c}_{lm} P_{lm}$.

Let the postage stamp rate for the loads be given by $k_{Load_T}^*$. Then

$$k_{Load_T}^* = \frac{\sum_{for\,all\,lm} P_{lm}\bar{c}_{lm}\alpha}{\sum_{i=1}^{n_L} P_{L_i}} \quad (29)$$

Let the postage stamp rate for the loads be given by $k_{Gen_T}^*$. Then

$$k_{Gen_T}^* = \frac{\sum_{\forall\ lm} P_{lm}\bar{c}_{lm}(1-\alpha)}{\sum_{k=1}^{n_G} P_{G_k}} \quad (30)$$

The aim of tracing compliant postage stamp method is to compute the closest traceable solution to the proportionate distribution of transmission system usage costs. Therefore, the objective function $f\{x,y\}$ in (1) is written as

$$f\{x,y\} = \sum_{i=1}^{n_L} \left|trprice_{Load_{pu}}^i - k_{Load_T}^*\right|$$

$$+ \sum_{k=1}^{n_G} \left|trprice_{Gen_{pu}}^k - k_{Gen_T}^*\right| \quad (31)$$

2) Objective Function for Loss Allocation: Another choice of objective function can be developed on similar lines for equitable distribution of losses. The per unit loss for load is given as follows:

$$loss_{pu}^{i} = (\sum_{k=1}^{n_G} y_k^i) - 1 \quad (32)$$

The per unit loss for generator is given as follows:

$$loss_{pu}^{k} = 1 - (\sum_{i=1}^{n_L} x_i^k) \quad (33)$$

Let us assume that percentage share of loss for generators in total loss of system is ShrLossG and percentage share of loss for loads in total loss of system is ShrLossL.

Therefore the objective function $f\{x,y\}$ in (1) for loss allocation is written as:

$$f\{x,y\} = \sum_{i=1}^{n_L} |loss_{pu}^{i} - k_{Load_l}^{*}| + \sum_{k=1}^{n_G} |loss_{pu}^{k} - k_{Gen_l}^{*}| \quad (34)$$

where $k_{Load_l}^{*}$ is the ratio of total system loss to total system load, and $k_{Gen_l}^{*}$ is the ratio of total system loss to total system generation.

3) Optimal Tracing Problem Formulation: Now, OPT problem that was defined in (1) can be explicitly expressed as follows:

$$min f(x,y) \quad (35)$$

$$\begin{bmatrix} A_d & 0 \\ 0 & A_u \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} b_d \\ b_u \end{bmatrix} \quad (36)$$

$$y_k^i P_{L_i} - x_i^k P_{G_k} \geq 0, \forall\, k \in \{1,2,\ldots,n_G\} \quad \forall\, i \in \{1,2,\ldots,n_L\} \quad (37)$$

$$[0,0,\ldots,0]^T \leq [x] \leq [1,1,\ldots,1]^T \; [y] \geq [0,0,\ldots,0]^T \quad (38)$$

where A_d is all the equality constrained matrices corresponding to generation tracing, and A_u is all the equality constrained matrices corresponding to load tracing. By solving above optimization problem we can allocate the generation and load to find the transmission usage cost and loss allocation.

III. Symbolic Reduction Technique

The tracing problem discussed in Section-II is modeled by a multi-commodity LP problem, where the number of variables is given by following equation:

$$No.ofVariable = n_b \times (n_G + n_L) + 2 \times (n_G \times n_L) + 2(n_G + n_L) \quad (39)$$

where n_G is number of generators, n_b is number of branches, and n_L is number of loads of the network. The number of variables required for different systems is tabulated in Table I.

Table I: No. of variable of different systems

Sr.No.	Name of the System	No. of Variables
1	IEEE 6 bus	70
2	IEEE 30 bus	1498
3	IEEE 57 bus	4640
4	IEEE 118 Bus	15252
5	NER Grid (India)	4511420

These numbers are quite large in comparison to the size of the system. In the final LP solution it is seen that many of the variables are set to zero. By identifying this large subset of zeros before the LP formulation we can reduce the size of the problem.

This suggests that additional work is required to improve computational efficiency of the method. This can be achieved by:

1. exploitation of sparsity linear programming and
2. improvement in modeling and algorithm.

The following methodology is proposed for the symbolic reduction. Symbolic reduction approach is a graph theoretic preprocessing on the output of load flow program to identify the reach of generator and load to transmission lines. It also identifies as to which generator can deliver power to loads and vice versa in the tracing frame work. The basic method can be explained as follows:

Let G be the directed graph of the system and arc be represented as $e(o,d)$. If load flow or state estimation output power flow is from node i to node j, then origin o is i and destination d is j. On the other hand if power flow is from j to i then origin o is j and destination d is i. If it is said that the generator at node k can reach the node q then the node q can be reached by traversing a sequence of connected arcs in the direction of flow. In other words, node q is in the reach of node k if following sequence of arcs $e(k,k1), e(k1,k2)\ldots\ldots e(kn,q)$ exists. Reach set of generator is the set of all nodes which can be reached by generator. A generator is said to reach an arc $e(l,m)$ if $l \in R_{gk}$ where R_{gk} is the reach set of the generator k. Similarly, the load reach set R_{li} can be found out and load i is said to reach an arc $e(l,m)$ if $m \in R_{li}$.

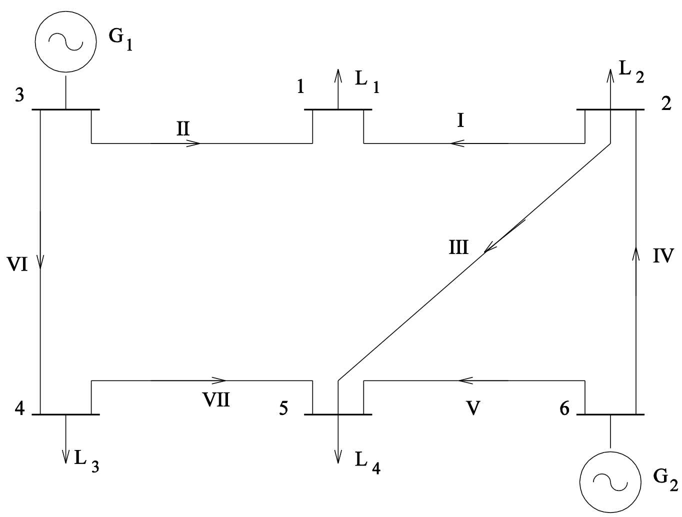

Fig. 2. Graph of a 6 bus system.

This method is illustrated using the Fig. 2 and 3. Let us take a directed graph as shown in Fig. 2 which is a 6-bus system with 2 generators, 4 loads, and 7 branches. If we start from generator G_1 and traverse in the direction of flow we can reach the nodes 1, 3, 4, and 5 only. So, the power from generator G_1 is flowing through branches II, VI, and VII only and it can supplying to the loads L_1, L_3, and L_4 only. This information is tabulated in Table II.

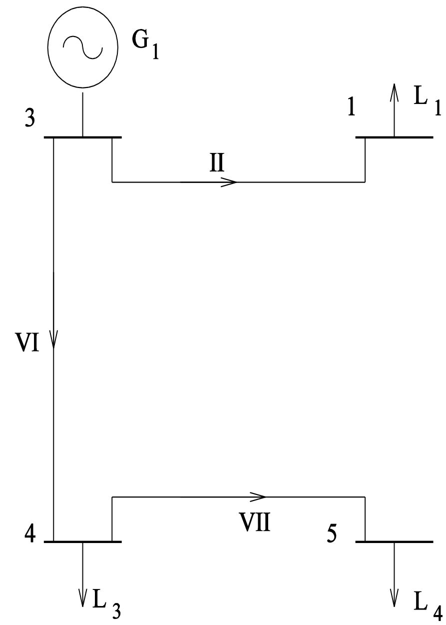

Fig. 3. Reach of Generator G1.

Table II: Reach set of generators

Generator	Branches	Loads
G1	II, VI, and VII	L1, L3 and L4
G2	I, III, IV, and V	L1, L2 and L4

Therefore instead of considering all the generators to decompose the flow of a branch, consider only those generators whose power is flowing through that branch. Then the equation (4) is modified as follows:

$$P_{lm} = \sum_{k \in R_{lm}^{G}} x_{lm}^{k} . P_{G_k}, \forall \text{ set of lines} \qquad (39)$$

where R_{lm}^{G} generator reach set for line lm i.e., it is the set of all generators which reach line lm.

Similarly, to decompose the load in terms of generator contributions consider the generators from which that load can be supplied. Then the equation (11) is modified as follows:

$$P_{L_i} = \sum_{k \in R_{L_i}^{G}} x_i^k . P_{G_k}, \forall\, i = 1, \ldots, n_L \qquad (40)$$

where $R_{L_i}^{G}$ generator reach set for load L_i i.e., it is the set of all generators which reach load L_i.

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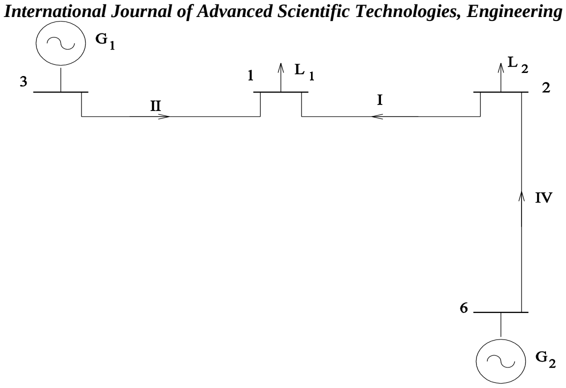

Fig. 4. Reach of load L1.

Table III: Reach set of loads

Load	Branches	Generators
L1	I,II, and IV	G1 and G2
L2	IV	G2
L3	VII	G1
L4	III, IV, V, VI, and VII	G1 and G2

From the graph shown in Fig. 4, load L_1 is drawing power from both the generators G_1 and G_2 through the branches I, II, and IV (for other loads it is listed in Table III). So, instead of decomposing the branch flow to all load components, we restrict decomposition to only those loads which can reach a line and generator. Thus, equations (6) and (13) can be modified as follows:

$$P_{lm} = \sum_{i\in R^L_{lm}}^{nlb_{lm}} y^i_{lm}.P_{L_i}, \forall set\, of\, lines \quad (41)$$

Where R^L_{lm} load reach set for line lm i.e., it is set of all loads which reach line lm.

$$P_{G_k} = \sum_{i\in R^L_{G_k}}^{nlg_i} y^i_k.P_{L_i}, \forall k = 1, \dots, n_G \quad (42)$$

where $R^L_{G_k}$ load reach set for generator G_k i.e., it is the set of all loads which reach generator G_k.

In this implementation to traverse the graph and keep the track of visited nodes and edges we used the well-known *depth first search* [6].

For an arc, if a generator or load is not in its reach set, then it implies that the corresponding commodity flow cannot reach the arc. Hence the corresponding generator tracing or load tracing variables can set to zero. This can achieve significant reduction in the number of variables actually modeled by LP. This leads to reduced number of variables in the LP problem. Further, the reduced problem can be solved by sparse LP method.

IV. Results

The algorithm has been tested on the standard IEEE test systems [8] and a real life Indian Grid system [9].

1. IEEE 6 Bus System
2. IEEE 30 Bus System
3. IEEE 58 Bus System
4. IEEE 118 Bus System
5. NER Grid (India) 488 Bus System [9].

The number of variables and the computational time of the algorithm with and without symbolic reduction technique is given in the Table IV. The table clearly shows that symbolic reduction technique significantly reduces number of variables and time required to solve optimal tracing problem. The number of variables for solving optimal tracing problem on NER Grid (India) system with symbolic reduction technique is 45703 and it is solved in 21.83 seconds. However, without application of the symbolic reduction technique, the number of variables is 4511420; a commercial optimization software could not solve the problem. The results of different systems is shown in Table IV. The percentage reduction in variables, constraints, and computational time with symbolic reduction technique is given in Table V and Fig. 5.

V. Conclusion

The number of variables and the time required for solving an optimal tracing problem have been reduced significantly with symbolic reduction technique. A commercial optimization tool box which failed to solve the large system like NER grid (India) system without symbolic reduction. However it is solved the same system in 21.83 seconds with symbolic reduction technique.

The symbolic reduction technique is not an addition to the optimal tracing problem, it is a prerequisite for the optimal tracing problem.

Table V: Comparison of existing and proposed methods

S.No	**System Name**	**No. of Buses**	**% Reduction**		
			Variables	**Constraints**	**Time**
1	6 Bus	6	41.43	29.71	30
2	30 Bus	30	63.82	52.02	40
3	57 Bus	57	75.63	63.86	61.11
4	118 Bus	118	86.37	75.03	79.29
5	NER Grid (India)	488	98.99	98.19	#

#-Could not solve optimal tracing problem without symbolic reduction technique

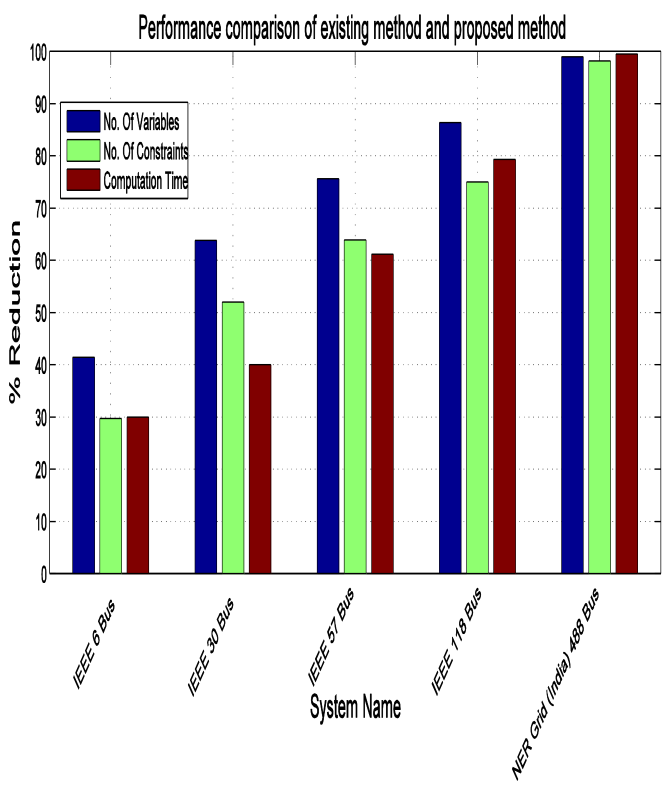

Fig. 5. Percentage reduction in variables, constraints and computational time using proposed method.

TABLE IV: COMPARISON OF ALGORITHMS WITH AND WITHOUT SYMBOLIC REDUCTION TECHNIQUE

S.No	System	No. of Nodes	No. of Variable		No. of constraints		Time required (secs)	
			Without reduction	With reduction	Without reduction	With reduction	Without reduction	With reduction
1	IEEE 6 Bus	6	70	41	488	343	0.01	0.007
2	IEEE 30 Bus	30	1498	542	1065	511	0.05	0.03
3	IEEE 57 Bus	58	4640	1131	3293	1190	0.18	0.07
4	IEEE 118 Bus	118	23544	3210	15252	3808	1.4	0.29
5	NER Grid (India)	488	4511420	45703	3995453	72168	#	21.83

- Could not solve optimal tracing problem without symbolic reduction technique

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