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# A Study on Hamilton Cycles of Rectangular Grid Graphs

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***Abstract*—Graph theory is one of the most vital twigs in mathematics and is expedient to locate relations between various allied systems. A Hamiltonian cycle in a graph G is a cycle that visits each vertex exactly once. In this paper we studied Hamilton cycles and spanning 2- connected subgraphs of rectangular grid graphs. This theory helps in grid computing such as improve efficiency by designating data and distribute it globally.**

***Index Terms*— Rectangular grid graphs, Hamilton Cycles, Spanning 2- connected subgraph.**

## I. INTRODUCTION

Determination of Hamilton cycles and triangles, the longest and shortest cycles in a graph attracts special attention. Hamiltonian cycles have been studied extensively in graph theory [6]. Thomassen [5] has studied the number of Hamilton cycles in tournaments. For complete graphs also this problem has been studied. We introduce the classes of grid graphs which we call truncated rectangular grid graphs.

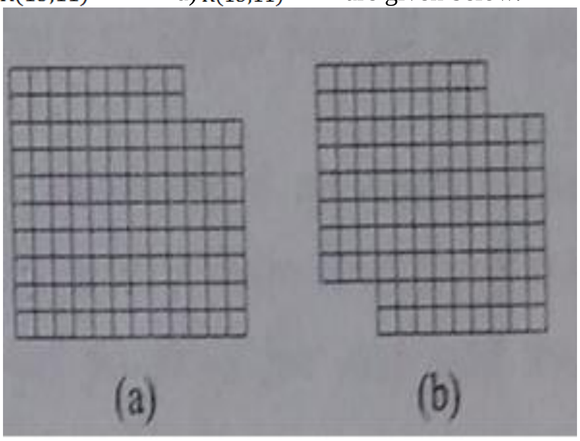
For $s \geq 3, t \geq 3, 0 \leq k \leq \min\{s-2, t-2\}$ and $0 \leq l \leq \min\{s-2, t-2\}$ . we define a 1 –corner truncated rectangular grid graph $R(s,t)^{-1(k,l)}$ as the subgraph obtained from $R(s,t)$ by deleting $k \times l$ vertices from one corner in $V(s,t)$ together with their incident edges in a natural drawing. For illustration, consider $R(13,11)^{-1(3,2)}$ in figure.2.1(a).

For $s \geq 6, t \geq 6, 1 \leq k \leq \min\left\{\frac{s-4}{2}, \frac{t-4}{2}\right\}$ and for $1 \leq l \leq \min\left\{\frac{s-4}{2}, \frac{t-4}{2}\right\}$ we define a 2-corner truncated rectangular grid graph $R(s,t)^{-2(k,l)}$ as the subgraph obtained from $R(s,t)$ by deleting $k \times l$ vertices from two opposite corners in $V(s,t)$ together with their incident edges in a natural drawing. For illustration, consider $R(13,11)^{-2(3,2)}$ in Figure 2.1 (b).

For $s \geq 6, t \geq 6, 1 \leq k \leq \min\left\{\frac{s-4}{2}, \frac{t-4}{2}\right\}$ and for $1 \leq l \leq \min\left\{\frac{s-4}{2}, \frac{t-4}{2}\right\}$ we define a 3- corner truncated rectangular grid graph $R(s,t)^{-3(k,l)}$ as the subgraph obtained from $R(s,t)$ by deleting $k \times l$ vertices from three corners in $V(s,t)$ together with their incident edges in a natural drawing. For illustration, consider $R(13,11)^{-3(3,2)}$ in Figure 2.1 (c).

For $s \geq 6, t \geq 6, 1 \leq k \leq \min\left\{\frac{s-4}{2}, \frac{t-4}{2}\right\}$ and $1 \leq l \leq \min\left\{\frac{s-4}{2}, \frac{t-4}{2}\right\}$ we define a 4- corner truncated rectangular grid graph $R(s,t)^{-4(k,l)}$ as the subgraph obtained from $R(s,t)$ by deleting $k \times l$ vertices from three corners in $V(s,t)$ together with their incident edges in a natural drawing. For illustration, consider $R(13,11)^{-4(3,2)}$ in Figure 2.1 (d).

Spanning 2- connected subgraphs with a minimum number of edges for the 1-corner truncated rectangular grid graph $R(s,t)^{-1(k,k)}$ and for the 4- corner truncated rectangular grid graph $R(s,t)^{-4(k,k)}$ were studied in [1]. Subsequently, in [2] these results were generalized to $R(s,t)^{-1(k,l)}$ and $R(s,t)^{-4(k,l)}$. In [4] we considered the other truncated rectangular grid graphs. We summarize the results in [3] and [4] in the Theorem 2.1. It characterizes which of the truncated rectangular grid graphs are Hamiltonian and guarantees the existence of a spanning 2-connected subgraph with at most three edges more than their number of vertices. Figure 2.1:Truncated rectangular grid graphs (a) $R(13,11)^{-1(3,2)}$ (b) $R(13,11)^{-2(3,2)}$ (c) $R(13,11)^{-3(3,2)}$ d) $R(13,11)^{-4(3,2)}$ are given below.



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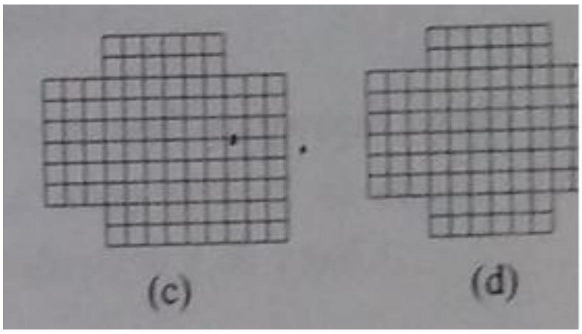



## II. MAIN RESULTS

**Theorem 2.1:** Let $R(s,t)^{-1(k,l)}$, $R(s,t)^{-2(k,l)}$ $R(s,t)^{-3(k,l)}$ and $R(s,t)^{-4(k,l)}$ denote the 1-corner truncated rectangular grid graph, the 2-corner truncated rectangular grid graph, the 3-corner truncated rectangular grid graph and the 4- corner truncated rectangular grid graph as defined above, respectively. Then:

a) $R(s,t)^{-1(k,l)}$ contains a spanning 2-connected subgraph with (at most) |V|+1 edges and is hamiltonian if and only if both s.l are even or both s .t and k.l are odd.

b) $R(s,t)^{-2(k,l)}$ contains a spanning 2-connected subgraph with
i) |V| edges if s.t is even and at least one of k and l is even if both s and t are even,
ii) |V|+2 edges if s and t are even and k and l are odd,
iii) |V|+1 edges in all other cases.
These number of edges are all best possible.

c) $R(s,t)^{-3(k,l)}$ contains a spanning 2-connected subgraph with
i) |V| edges if both s.t and k.l are even,
ii) |V|+2 edges if all of s, t, k and l are odd,
iii) |V|+1 edges in all other cases.
These number of edges are all best possible.

d) $R(s,t)^{-4(k,l)}$ contains a spanning 2-connected subgraph with (at most)
i) |V|+3 edges and is Hamiltonian if and only if s.t is even.
ii) The bound |V|+3 is best possible for any odd numbers s, t, k and l.

**Proof:** We need the following lemma to prove the result.

**Lemma 2.2:** Suppose a planar graph G has a Hamilton cycle H. Let G be drawn in the plane and let $r_i$ denote the number of faces inside H bounded by i edges in this planar embedding. Let $r_i'$ be the number of faces outside H bounded by i edges. Then the numbers $r_i$ and $r_i'$ satisfy the following equation $\sum_i (i-2)(r_i - r_i') = 0$.

We use this lemma to show that $R(s,t)^{-1(k,l)}$ and $R(s,t)^{-3(k,l)}$ contain no Hamilton cycle if $s \cdot t$ and $k \cdot l$. have a different parity, and that $R(s,t)^{-2(k,l)}$ and $R(s,t)^{-4(k,l)}$ contain no Hamilton cycle if $s \cdot t$ is odd.

**Corollary 2.3:** $R(s,t)^{-j(k,l)}$ contains no Hamilton cycle if (s.t and k.l have a different parity For j=1 or 3) or (s.t is odd for j=2 or 4)

Proof : There is exactly one face with 2(s+t-2) edges and there are exactly (s-1)(t-1)-j.k.l

Faces with four edges in the planar (natural) drawing of the j- corner truncated rectangular grid graph $R(s,t)^{-j(k,l)}$ for j= 1, 2, 3 or 4.

Let this graph be Hamiltonian. Then by Lemma 2.2, we have $(2(s+t-2)-2)(-1)+(4-2)(r_4 - r_4') = 0$

Hence $r_4 + r_4' = s+t-3$. It is easy to check that the number of faces with four edges is $r_4 + r_4' = (s-1)(t-1) - j.k.l.$

From equation (2.1) and (2.2) we obtain $2r_4 = s.t - j.k.l - 2$

It implies that either $s.t$ and $k.l$ are even or $s.t$ and $k.l$ are odd for $j = 1$ or 3, and that $s.t$ is even for$j = 2$ or 4.

Example . Consider $R(7,8)^{-1(2,1)}$ contains no Hamilton cycle.

For other values of $s, t, k$and $l$, it is not difficult to see, from the patterns in the figures that now follow, has to extend the solutions.

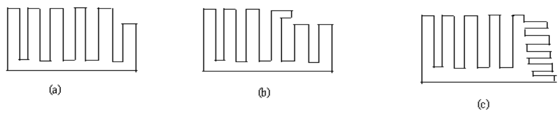



Figure 2.2: Hamilton cycles for (a) $R(12,11)^{-1(2,3)}$ b) $R(12,11)^{-1(3,2)}$ c) $R(12,11)^{-1(3,1)}$

→ A Hamilton cycle for $R(12,11)^{-1(2,3)}$ is shown in Figure 2.2 (a). The pattern in this figure can be used for finding a Hamilton cycle for the 1- corner truncated rectangular grid graph for either (any numbers $t$ and $l$, and any even numbers $s$ and $k$) or (any number $s$ , $k$ and any even numbers $t$ and $l$).

Example: Hamilton cycle for $R(10,11)^{-1(2,3)}$

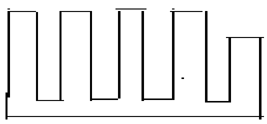


Figure 2.2 (a)

In Figure 2.2 (b) we show a Hamilton cycle for $R(12,11)^{-1(3,2)}$. The pattern in Figure 2.2 (b) can be used for finding a Hamilton cycle for the 1-corner truncated rectangular grid graph for either (any number $t$, any even numbers $s$ and $l$, and any odd number $k$) or (any number $s$, any even numbers $t$ and $k$, and any odd number $l$).

Example: Hamilton cycle for $R(10,8)^{-1(1,2)}$

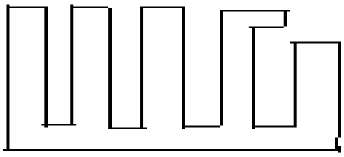
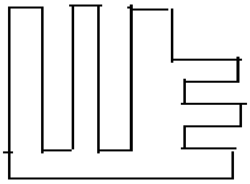
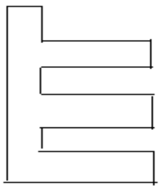


Figure 2.2 (b)

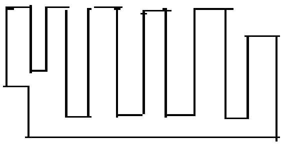
Meanwhile, in Figure 2.2(c) we show a Hamilton cycle for $R(12,11)^{-1(3,1)}$ . The pattern in Figure 2.2 (c) can be used for finding a Hamilton cycle for the 1-corner truncated rectangular grid graph for any odd numbers $s, t, k$ and $l$.

Example: Hamilton cycle for $R(7,9)^{-1(1,3)}$ and Hamilton cycle for $R(7,5)^{-1(3,1)}$



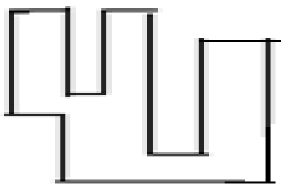


→ A Hamilton cycle for $R(12,11)^{-2(2,3)}$ is shown in Figure 2,4(a). The pattern's in this figure can be used for finding a Hamilton cycle for the 2- corner truncated rectangular grid graph for either (any numbers $t$ and $l$ and any even numbers s and k)

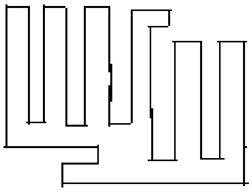


(a)

Example: Hamilton cycle for $R(6,8)^{-2(2,1)}$

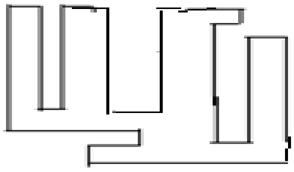


In Figure 2.4(b) we show a Hamilton cycle for $R(12,11)^{-2(3,2)}$. The pattern in Figure 2.4(b) can be used for finding a Hamilton cycle for the 2-corner truncated rectangular grid graph for either (any even numbers $s$ and $l$, and any odd numbers $t$ and $k$) or (any even numbers $t$ and $k$, and any odd numbers $s$ and $l$).

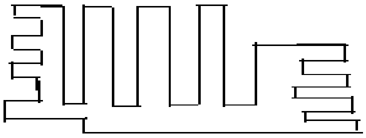


(b)

Example: Hamilton cycle for $R(8,7)^{-2(1,2)}$

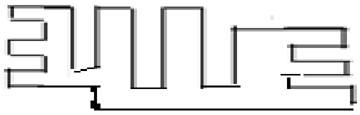


In Figure 2.4(c) we show a Hamilton cycle for $R(12,11)^{-2(3,3)}$. The pattern in Figure 2.4(c) can be used for finding a Hamilton cycle for the 2- corner truncated rectangular grid graph for either (any even number $s$, and any odd numbers $t, k$ and $l$) or (any even number $t$, and any odd numbers $s, k$ and $l$).



(c)

Example: Hamilton cycle for $R(10,7)^{-2(3,1)}$



→ Hamilton cycles for $R(12,11)^{-3(2,3)}$, $R(12,11)^{-3(3,2)}$ are shown in Figure 2.6. The pattern in Figure 2.6(a) can be used for finding a Hamilton cycle for the 3- corner truncated rectangular grid graph for either (any numbers $t$ and $l$, and any even numbers $s$ and $k$) or (any numbers $s$ and $k$, and any even numbers $t$ and $l$). The Pattern in Figure 2.6(b) can be used for finding a Hamilton cycle for the 3-corner truncated rectangular grid graph for either (any even numbers $s$ and l, and any odd numbers t and k) or (any even numbers t and k, any odd numbers s and l).

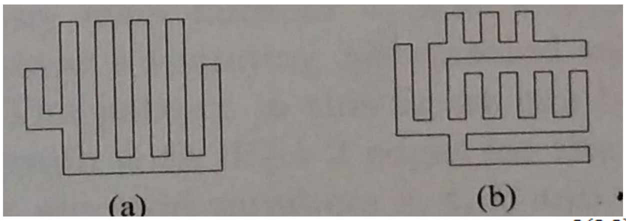



Figure 2.6: Hamilton cycles for (a) $R(12,11)^{-3(2,3)}$ (b) $R(12,11)^{-3(3,2)}$

→ Hamilton cycles for $R(12,11)^{-4(2,3)}$, $R(12,11)^{-4(3,2)}$ are shown in Figure 2.7. The pattern in Figure 2.7(a) can be used for finding a Hamilton cycle for the 4- corner truncated rectangular grid graph for either (any numbers $t$and $l$, and any even numbers $s$ and $k$) or (any numbers $s$ and $k$, and any even numbers $t$ and $l$). The Pattern in Figure 2.7(b) can be used for finding a Hamilton cycle for the 4-corner truncated rectangular grid graph for either (any numbers $t$ and $l$, and any even number $s$ and any odd number $k$) or (any numbers $s$ and $k$, any even number $t$, and any odd number $l$).

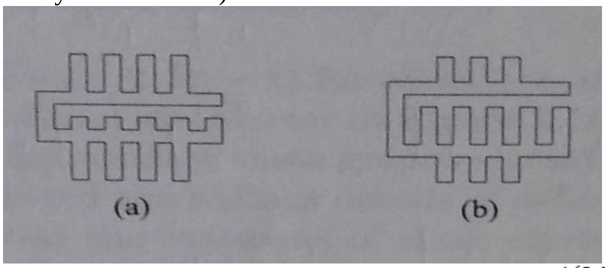



Figure 2.7: Hamilton cycles for (a) $R(12,11)^{-4(2,3)}$ (b) $R(12,11)^{-4(3,2)}$

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