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Numerical Analysis of boundary layer flow of non-Newtonian fluid in porous media

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Abstract:

The unsteady free convective heat and mass transfer in a non-Newtonian Walters-B viscoelastic fluid past a vertical cone with radiation in a non-Darcian isotropic porous regime is presented. The dimensionless unsteady, coupled and non-linear partial differential conservation equations for the boundary layer regime are solved by the implicit finite difference scheme. The velocity, temperature and concentration fields have been studied for the effect of Darcy number, radiation parameter, viscoelasticity parameter. The local skin-friction Nusselt number and Sherwood number are also analyzed. It is observed that the flow is accelerated for higher Da values causing an increase in the velocity. The present results are compared with available results in literature and are found to be in good agreement.

Key words: viscoelasticity; porous medium; finite difference method;

I. INTRODUCTION:

Combined heat and mass transfer in fluid-saturated porous media finds applications in a variety of engineering processes such as heat exchanger devices, petroleum reservoirs, chemical catalytic reactors and processes and others. Sivaraj and Rushi Kumar [7] analyze the effects of non-uniform heat source/sink and higher order chemical reaction on unsteady, free convective viscoelastic fluid flow over a moving vertical cone and a flat plate saturated with porous medium.

The above studies did not consider combined viscoelastic, heat and mass transfer past a vertical cone in the presence of thermal radiation in a non-Darcian isotropic porous regime. Owing to the significance of this problem in chemical technological processing the unsteady natural convection heat and mass transfer from a vertical cone is considered.

II. MATHEMATICAL FORMULATION:

An axi-symmetric transient natural convective heat and mass transfer in a viscoelastic fluid from an isothermal vertical cone with uniform surface temperature and concentration in a Darcy-Forchheimer fluid saturated isotropic porous medium in the presence of radiation is considered. The Boussinesq's approximation is taken into account for the buoyancy effects induced by thermal and mass diffusion. The co-ordinate system chosen (as shown in Fig.1) is such

that the x- coordinate is directed along the surface of the cone from the apex (x = 0) and the y- coordinate is orientated perpendicular to this i.e. at right angles to the cone surface, outwards. Here, ϕ designates the semi-vertical angle of the cone and r is the local radius of the cone.

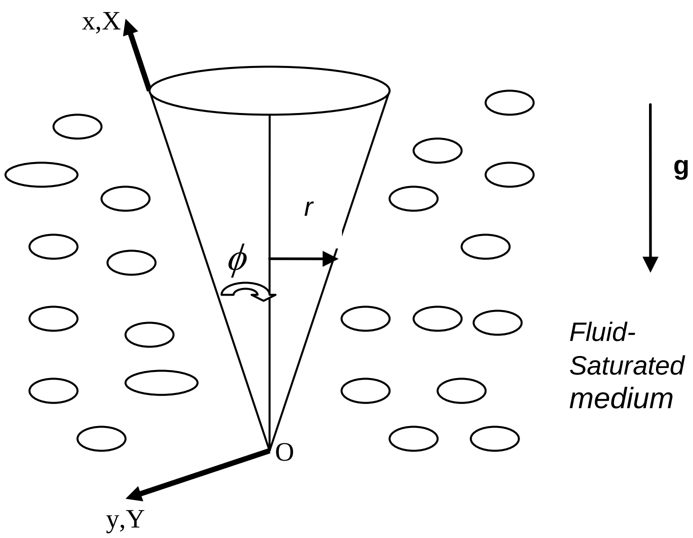

Figure 1: Physical Model

Under the above assumptions, implementing the shear-stress strain tensor for a Walters-B liquid, the appropriate unsteady incompressible conservation equations for the regime may be shown to take the form:
Proceeding with analysis, we implement the following non-dimensional quantities:

$$X=\frac{x}{L},\quad Y=\frac{y}{L}(Gr_L)^{\frac{1}{4}},\quad R=\frac{r}{L},\text{ where}$$
$$r=x\sin\phi, V=\frac{vL}{\nu}(Gr_L)^{\frac{-1}{4}}, U=\frac{uL}{\nu}(Gr_L)^{\frac{-1}{2}},$$
$$t=\frac{\nu t'}{L^2}(Gr_L)^{\frac{1}{2}},\ Da=\frac{K}{L^2} \tag{1}$$
$$T=\frac{T'-T'_\infty}{T'_w(L)-T'_\infty}, Gr_L=\frac{g\beta\left(T'_w(L)-T'_\infty\right)L^3}{\nu^2}$$
$$\Pr=\frac{\nu}{\alpha},\quad F=\frac{k_e k}{4\sigma_s T'^3_\infty}, C=\frac{C'-C'_\infty}{C'_w-C'_\infty}, M=\frac{\sigma_0 B_0^2 L}{\rho}$$
$$,\Gamma=\frac{k_0 Gr_L^{\frac{1}{2}}}{L^2}, N=\frac{\beta^*(C'_w-C'_\infty)}{\beta(T'_w-T'_\infty)}, Sc=\frac{\nu}{D},\ Fs=\frac{b}{L}$$

Governing Equations in the non-dimensional form are

$$\frac{\partial(UR)}{\partial X}+\frac{\partial(VR)}{\partial Y}=0 \tag{2}$$
$$\frac{\partial U}{\partial t}+U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}$$
$$=\frac{\partial^2 U}{\partial Y^2}-\Gamma\frac{\partial^3 U}{\partial Y^2\partial t}-MU^2+T\cos\phi+NC\cos\phi-\frac{U}{DaGr_L}-\frac{Fs}{Da}U^2 \tag{3}$$
$$\frac{\partial T}{\partial t}+U\frac{\partial T}{\partial X}+V\frac{\partial T}{\partial Y}=\frac{1}{\Pr}\left[1+\frac{4}{3F}\right]\frac{\partial^2 T}{\partial Y^2} \tag{4}$$
$$\frac{\partial C}{\partial t}+U\frac{\partial C}{\partial X}+V\frac{\partial C}{\partial Y}=\frac{1}{Sc}\frac{\partial^2 C}{\partial Y^2} \tag{5}$$

The corresponding initial and boundary conditions are

$$t\le 0: U=0, V=0, T=0, C=0 \text{ for all X, Y,}$$
$$t>0: U=0, V=0, T=1, C=1 \text{ at } Y=0, \tag{6}$$
$$U=0,\ T=0,\ C=0 \quad \text{at } X=0,$$
$$U\to 0, T\to 0,\ C\to 0 \text{ as } Y\to\infty.$$

The dimensionless local values of the skin friction (surface shear stress), the Nusselt number (surface heat transfer gradient) and the Sherwood number (surface concentration gradient) are given by the following expressions:

$$\tau_x=-\left(\frac{\partial U}{\partial Y}\right)_{Y=0} \tag{7}$$
$$Nu_x=-X\left(\frac{\partial T}{\partial Y}\right)_{Y=0} \tag{8}$$
$$Sh_x=-X\left(\frac{\partial C}{\partial Y}\right)_{Y=0} \tag{9}$$

III. NUMERICAL SOLUTION:

The transient, non-linear equations (2)–(5) under the conditions (6) are solved by an implicit finite difference scheme of Crank-Nicolson type which is discussed in [1],[3],[4] and [6]. The dimensionless governing equations are reduced to tri-diagonal system of equations and solved by Thomas algorithm as discussed in [2]. The region of integration is considered as a rectangle with $X_{\max}=1$ and $Y_{\max}=22$ where $Y_{\max}$ corresponds to $Y=\infty$ which lies very well outside both the momentum and thermal boundary layers. The maximum of Y was chosen as 22, after some preliminary investigation so that the last two boundary conditions are satisfied within the tolerance limit 10^{-5}. The mesh sizes have been fixed as $\Delta X=0.05, \Delta Y=0.05$ with time step $\Delta t=0.01$. The computer program of the algorithm is executed in FORTRAN running on a PC. The scheme is unconditionally stable. The local truncation error is $O(\Delta t^2+\Delta Y^2+\Delta X)$ and it tends to zero as $\Delta t, \Delta X$ and ΔY tend to zero. Hence, the scheme is compatible. Stability and compatibility ensure the convergence.

IV. RESULTS AND DISCUSSION:

Only selective figures have been reproduced here for brevity. In order to prove the accuracy of the computations in steady state at X = 1.0, Pr = 0.7, $\eta=Y$ and considering $Gr_L^*=Gr_L cos\phi=\frac{g\beta cos\phi\left(T'_w-T'_\infty\right)L^3}{\nu^2}$ are compared with available similarity solutions in the literature. The local Nusselt number Nu_X values for different Prandtl number are compared with the results of isothermal case (n = 0) of [5] and [6], and are found to be in good agreement.

Figures 1(a) and 1(b) show the influence of radiation parameter, *F*, on steady state velocity (*U*) and temperature (*T*) distributions with distance into the boundary layer, transverse to the cone surface (*Y*). An increase in *F* from 0 through 0.5, 1.0, 3.0, 5.0, 10.0 to 100.0, causes a significant reduction in velocity with distance into the boundary layer i.e. retards the flow.

Figures 2(a) and 2(b) show the effect of Darcy number (Da) on dimensionless velocity (U) and temperature (T) with transformed radial coordinate (Y) close to the leading edge (i.e. cone apex) at X = 1.0. To study the influence of regime permeability from sparsely packed media to densely packed materials the following values Da = 1.0, 0.1, 0.01, 0.001 are considered.

It is observed that the steady state temperature (*T*) distributions to different values of the viscoelastic material parameter Γ from 0 to 0.001, 0.003 and to the largest value of 0.005, decrease the temperature distributions accompanies an increase in Γ.

V. CONCLUSIONS:

The unsteady natural convective heat and mass transfer in a viscoelastic fluid past a vertical cone with radiation in a non-Darcian isotropic porous regime is considered. The dimensionless conservation equations for the boundary layer regime are solved by the implicit finite difference scheme of Crank-Nicolson type. It is observed that the flow is accelerated for higher Da values causing an increase in the velocity. Velocity boundary layer thickness will be increased with a rise in Da and thermal boundary layer thickness reduced. The increasing viscoelasticity accelerates the streamwise velocity and shear stress (local skin friction), local Nusselt number and local Sherwood number. Decreasing *F* accelerates the flow and increases temperatures in the boundary layer regime.

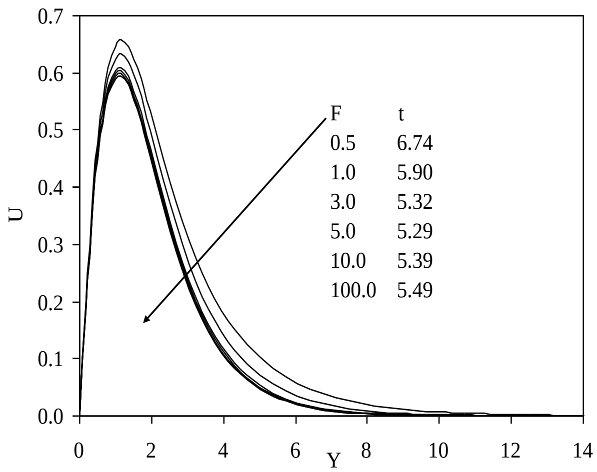

Fig.1(a).Steady state velocity profiles at X=1.0 for different F

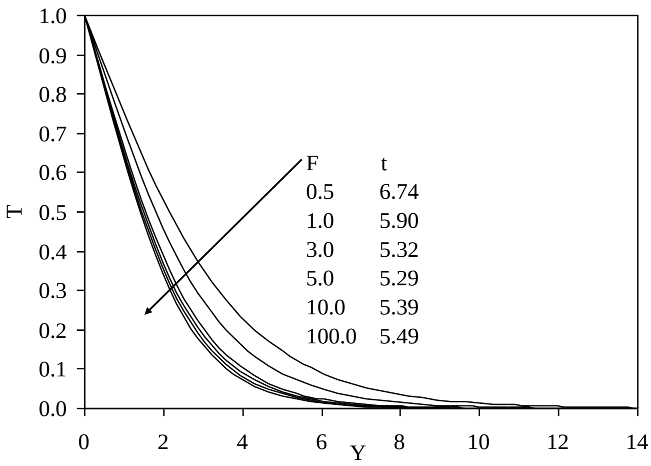

Fig.1(b).Steady state temperature profiles at X=1.0 for different F

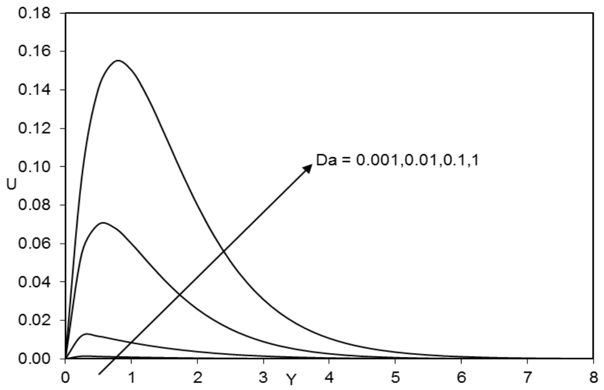

Fig.2(a).Steady state velocity profiles for different Da at X = 1.0

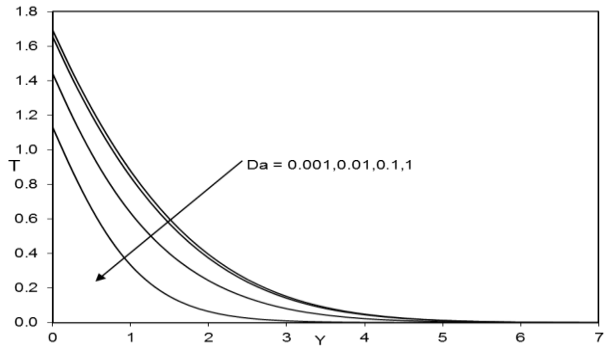

Fig.2(b).Steady state temperature profiles for different Da at X = 1.0

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