International Journal of Advanced Scientific Technologies, Engineering and Management Sciences (IJASTEMS-ISSN: 2454-356X) Volume.3,Special Issue.1,March.2017

VERTEX DOMINATION OF QUADRATIC RESIDUE CAYLEY GRAPHS

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Abstract*—Graph Theory has been actualize as one of the most advantageous branches of mathematics and attain applications in all most all branches of sciences. The Quadratic residue Cayley graph denoted by G(Zp, Q) is the Cayley graph associated with the quadratic residue function. In this paper we study vertex cover and vertex covering number of G(Zp, Q) by developing an algorithm. The theory of vertex domination helps in finding inner alignments of a task and progresses the facility of a task in connected systems such as communications and networking.

***Index Terms*— Quadratic residue Cayley graph, Vertex cover, Vertex covering number.**

I. INTRODUCTION

Domination in graphs is a booming area of research at present which provides applications, such as networks design, mobile computing and telecommunication etc. Cayley graphs are excellent representations for interconnection networks with parallel processing and distributed calculation. Allan[1], Cockayne[2] have studied various domination parameters of graphs. Nathanson[5] was the pioneer in introducing the concepts of Number Theory, particularly, the 'Theory of Congruences' in Graph Theory, thus paved the way for the emergence of a new class of graphs, namely "Arithmetic Graphs".

Cayley Graphs are another class of graphs associated with elements of a group. If this group is associated with some Arithmetic function then the Cayley graph becomes an Arithmetic graph. The Cayley graph associated with the quadratic residue function is called Quadratic residue Cayley graph. Jeelanibegum[3] have studied edge domination of Quadratic residue Cayley graphs.

II. TERMINOLOGY

Let p be an odd prime and n be a positive integer such that n $\not\equiv$ 0 (mod p). If the quadratic congruence $x^2 \equiv n$ (mod p) has a solution then n is called a quadratic residue modulo p. Let S be the set of quadratic residues modulo p and let S*={s, p-s / s$\in$ S, s $\neq$ p}, where p is an odd prime. The Quadratic residue Cayley graph G(Z_p, Q) is defined as the graph whose vertex set is V= Z_p ={0,1,2,3,…..,(p-1)} and the edge set is E={(x, y)/x - y or y - x is in S*}.

The graph G(Z_{13}, Q) is given below in figure 2.1.

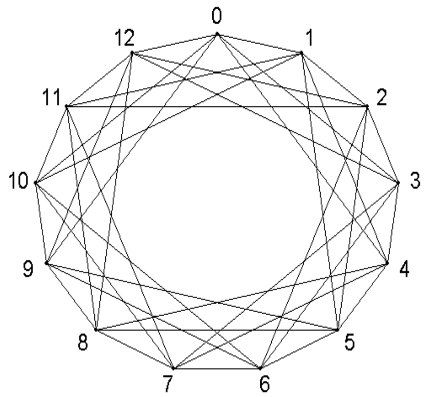

Figure 2.1 G(Z_{13}, Q)

Lemma 2.1[4]: The number of edges in G(Z_p, Q) is $\frac{|Z_p||S^*|}{2}, |S^*| - regular.$

Theorem 2.2[4]: The graph G (Z_p, Q) is complete if and only if, p $\nmid$ (a2+b2), for any positive integers a & b.

Theorem 2.3[4]: The graph G (Z_p , Q) is complete if p is of the form 4m + 3.

Theorem 2.4[4]: If p is of the form 4m + 1, then the sets Q and S* are the same, so that the graph G (Z_p , Q) is not complete.

A set S ⊂ V(G) is said to be a vertex covering set or vertex cover of the graph G if every edge has at least one end point in S.
A vertex covering set with minimum cardinality is called minimum vertex covering set or minimum vertex cover.

The vertex covering number of the graph G is the cardinality of any minimum vertex covering set of the graph G. It is denoted by $\beta(G)$.

III. ALGORITHM

Step 1: Consider the graph G(Z_p, Q). Find quadratic residue set Q and compute S*={q, p - q / q∈ Q, q ≠ p}
Step 2: Let vertex set V = {0,1,2,---,(p-1)} and Edge set E={x-y or y-x S*/ x,y ∉ V}.
Step 3: Assume V1 = V, then V1 becomes a vertex cover.
Let X = ϕ and C = p.
Step 4: Select a vertex s, which is not adjacent with the vertices of V1 – X. If such s does not exists then go to step 7.
Step 5: Remove vertex s from V1 - X. Set X=X ∪ {s}, C = C-1
Step 6: If vertices of V1 – X covers all the edges of G(Zp, Q), then V1 – X becomes a vertex cover.
Then go to step 4.
Step 7: The set V1 – X becomes a minimum vertex cover, and C gives the vertex covering number. STOP.
We now illustrate the algorithm as follows .
Consider the graph G(Z_{13}, Q), which is a 6-regular.
The quadratic residue set is Q = {1,3,4,9,10,12} and S*={q, p-q / q∈ Q, q ≠ p}= Q.
Here the vertex set V={0,1,2,---,12} and Edge set E={x-y or y-x S*/ x,y∉ V}.
Let the vertex covering set is V1 =V, that contains all vertices of G(Z13, Q).

i.e., V1=V={0,1,---,12} becomes a vertex cover.
Suppose X=ϕ , C =13. Suppose that the vertex s = 0 is removed from V1 – X.
We have X={0} C = 11and V1 – X ={1,2,---,12}, this set covers all the edges in G(Z13, Q).
It is a vertex cover. As V1 is not a Minimum vertex covering.
We now remove a vertex that vertex is not adjacent to any vertices of X.
That is 0 is not adjacent to 2,5,6,7,8,11.
In these vertices, we select a vertex s = 2.
Then we get X={0,2} , C =11 and
V1 – X = {1,3,4,5,6,7,8,9,10,11,12} is also a vertex covering set Now we Further we remove a vertex i.e., 0 & 2 are not adjacent to 7,8.
In these vertices, we select a vertex is s = 7.
Then we get X={0,2,7} , C = 10 and
V1 – X = {1,3,4,5,6,7,9,10,11,12} is also a vertex covering set & V3 is not a minimum vertex covering set.
Finally, there is no vertex s found that is not adjacent to X.
Then we stop the process and conclude that
V1 – X = {1,3,4,5,6,7,9,10,11,12} becomes a minimum vertex covering. The covering number of G(Z_{13}, Q) is 10.

IV. CONCLUSION

In this paper we present an algorithm to find vertex cover which is minimum and obtained vertex covering numbers of G(Z_p, Q), for p = 5, 7, 13, 17, 29, 37.

TABLE 1.1

G(Z_p,Q)	Minimum Vertex Covering Set	Vertex covering number
P=5	{1,3,4}	3
P=7	{1,2,3,4,5,6}	6
P=13	{1,3,4,5,6,7,9,10,11,12}	10
P=17	{1,2,3,4,6,7,8,9,11,12,13,14,15,16}	14
P=29	{1,2,3,4,5,6,7,9,10,11,12,13,14,15, 16,17,19,20,21,22,23,24,25,27,28}	25
P=37	{1,2,3,4,5,6,7,9,10,11,12,13, 15,16,17,18,19,20,21,23,24,25, 26,27,28,29,30,31,32,33,34, 35,36}	33

Further we would like to develop results on these minimum vertex covering sets and vertex covering number of G(Z_p, Q).

ACKNOWLEDGEMENT

The research was supported by DST, New Delhi. The first author is thankful to DST [Ref: No.SR/WOS-A/MS-07/2014 (G)] New Delhi and management of MITS, Madanapalle.

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