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INVERSE DOMINATION OF DIVISOR CAYLEY GRAPHS

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Abstract— ***Let $D \subset V$ be the dominating set of $G(V,E)$. A nonempty subset D' of the vertex set V - D is an inverse dominating set of G, if every vertex in $V'-D'$ is adjacent to at least one vertex in D' , where $V'=V-D$. The Cayley graph of $G(Z_n,D)$, associated with the set S* is called the divisor Cayley graph and it is denoted by $G(Z_n,D)$, where S*= {s, n- s / s ∈ S, n ≠ s} and S be the set of divisors of n, ($n \geq 1$) . That is, $G(Z_n,D)$ is the graph whose vertex set is V ={0,1,2,…,(n-1)} and the edge set is E = {(x , y) / x - y or y - x is in S*}. In this paper we study inverse dominating sets and obtain inverse domination number of $G(Z_n,D)$. This theory helps in finding inner alignments of tasks and progresses the facility of a task in connected systems such as communications and networking.***

Index Terms— ***Divisor Cayley graph, Inverse Dominating Sets, Inverse Domination Number.***

I. INTRODUCTION

Allan[1], Cockayne[2] have studied various domination parameters of graphs. Kulli[5, 6] first introduced the concept of Inverse domination and Inverse total domination in graphs. Domke[4] have studied the inverse domination number of a graph. Here are some of the properties of divisor Cayley graphs.

THEOREM 1.1[3]: The graph $G(Z_n,D)$ is |S*| - regular, and the number of edges in $G(Z_n,D)$ is $\frac{n|S^*|}{2}$.

THEOREM 1.2[3]: The graph $G(Z_n,D)$ is Hamiltonian and hence connected .

THEOREM 1.3[3]: (a) Degree of each vertex in $G(Z_n,D)$ is odd and if, and only if, n is even.

(b) Degree of each vertex in $G(Z_n,D)$ is even and if, and only if, n is odd.

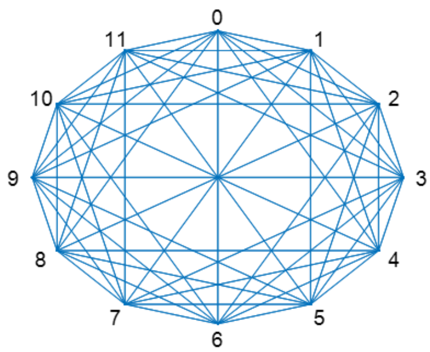
THEOREM 1.4[3]: The graph $G(Z_n,D)$ is not bipartite.

THEOREM 1.5[3]: If n is a prime, then the graph $G(Z_n,D)$ is the outer Hamilton cycle.

II. INVERSE DOMINATION

A nonempty subset D of V is a dominating set, if every vertex in V-D is adjacent to at least one vertex in D and the domination number is denoted by γ. Let D be the minimum dominating set, if V-D contains a dominating set D', then D' is called an Inverse Dominating set with respect to the Dominating set D. The minimum cardinality of D' is called the Inverse Domination number and it is denoted by γ'.

The graph $G(Z_{12},D)$ is given below.



III. MAIN RESULTS

Here we propose an algorithm that possess minimal inverse dominating sets of $G(Z_n,D)$, from which we can find inverse domination number.

ALGORITHM 3.1:

Step 1: Consider the graph $G(Z_n,D)$, with the vertex set $V' = V-D_i$.

we choose every D_i is a minimal dominating set of $G(Z_n,D)$.

Assume m=2, b=0, z=0, X=Ø, Q=Ø, L=Ø, M=Ø, R=Ø, u=v=0, n=|V|

//Finding Inverse Dominating Sets //

Step 2: Write all m- combination sets of V-D.

i.e., $^{(n-|D|)}C_m$ subsets of V-D.
Denote the collection of these sets by X. Let u = |X|.
Step 3: If u=0 then go to step 12, otherwise go to step 4.
Step 4: Find X – L , Select a set D' of X – L.
Find N[D']. Place D' in L.
Step 5: If then go to step 6, Otherwise u=u-1, go to step 3.
Step 6: Print "D' is a Inverse Dominating set".
Find proper subsets of D'. Denote the collection of these sets by Q.
Let v = |Q|.

// Finding Minimal Inverse Dominating Sets //

Step 7: If v = 0, go to step 11, otherwise go to step 8.
Step 8: Find Q-M.
Select a set A of Q-M.
Find N[A].
Place A in M.
Step 9: If $\left(V' - A\right) \subset N[A]$ then A is a inverse dominating set.
That is D' is not a minimal inverse dominating set.
Put u=u-1, go to step 3.
Else go to step 10.
Step 10: Put v=v-1, go to step 7.
Step 11: Write "D' is a minimal inverse dominating set".
Put b = b + 1. Place the set D' in R.
// R is the set of all minimal inverse dominating sets //
Put u = u-1. Let z = |D'|. Go to step 3.
Step 12: If z≠0 then Stop. Else m=m+1
And Let L=Ø, X=Ø, Q=Ø, M=Ø. Go to step 2.

THEOREM 3.2: In $G\left(Z_n, D\right)$, the following hold.

(i) $\gamma'(G(Z_n, D)) = z$,
(ii) The number of minimal total dominating sets is b,

Proof:

(i) From the above algorithm, we have R, the set of all minimal inverse dominating sets. And the cardinality of every minimal inverse dominating set z. Thus the minimum cardinality of the sets in R is also z. Therefore $\gamma'(G(Z_n, D)) = z$
(ii) By the algorithm, we have |R|, the number of minimal inverse dominating sets which is equal to b. Thus the number of minimal inverse dominating sets in $G\left(Z_n, D\right)$ is b.

For example consider the graph $G\left(Z_n, D\right)$ for n = 5, which is a outer Hamilton cycle.
Then S*={1,4}. The vertex set of $G\left(Z_5, D\right)$ is V={0,1,2,3,4} and edge set is E={(0,1), (1,2), (2,3), (3,4), (4,0)}. First we find the minimal dominating sets of $G\left(Z_n, D\right)$, then the possible minimal dominating sets of are D1 = {0,2}, D2 = {0,3}, D3 ={1,3}, D4 ={1,4},
D5 ={2,4}. As we have the minimal dominating sets in $G\left(Z_n, D\right)$, we can find inverse dominating sets of $G\left(Z_n, D\right)$. Consider D ={0,2} is a minimum dominating set. Then number of vertices in D is 2. Then there exists an inverse dominating set D' of with the vertex set is V-D ={1, 3, 4}, the inverse dominating sets corresponding to D={0,2} are {1,4} or {1,3}. Here D' = {1, 4},1 is isolated vertex and 4 is adjacent to 3. The set D' satisfies the domination property in V-D. Thus, {1,4} is an inverse dominating set correspond to {0, 2}.
Also, the inverse dominating sets corresponding to {0,3}, {1,3}, {1,4}, {2,4} are
D= {0, 3} then the IDS is D' = {1,4} or {2,4},
D= {1, 3} then the IDS is D'= {0,2} or {4,2},
D= {1, 4} then the IDS is D'= {0,2} or {0,3},
D= {2, 4} then the IDS is D'= {0,3} or {1,3}. Therefore it follows that the minimum dominating sets Di, i=1,2,...,5 are the minimum inverse dominating sets corresponding to their minimum dominating sets. The cardinally taken over all minimum inverse dominating sets is 2.

RESULTS AND DISCUSSION 3.3:

For n = 2, 3, 4, 6, the symmetric set s*= {1, 2 ,... . . . . . ., (n – 1)}. Since each of these graphs is |S*|- regular. This shows that each vertex of $G\left(Z_n, D\right)$ is adjacent to all other vertices of $G\left(Z_n, D\right)$. So $G\left(Z_n, D\right)$ is a complete graph for n = 2, 3, 4, 6. In complete graphs the dominating set D contains any single vertex set corresponding to the set V. Now the inverse dominating set D' also contains any single vertex set except the vertices in D. i.e. D' is sub set of V-D.
For example we consider the graph $G\left(Z_6, D\right)$, is a complete graph. In this graph the dominating set D= {0} then V-D={1,2,3,4,5}. Now the inverse dominating set D'={1} corresponding to D. As every vertex forms a minimum inverse dominating set in V-D. It follows that $\gamma'(G\left(Z_n, D\right)) = 1$.

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