Experimental and analytical study for the Radial Variation of Viscosity of non-Newtonian behavior of blood flow through overlapping stenosis: Computational Modeling Approach

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Abstract:
A comparative study of experimental and analytical analysis has been carried out for the non-Newtonian behavior of blood flow through overlapping stenosis considering radial variation of viscosity. In this, the constitutive equations of the model are solved analytically using the initial and boundary conditions to get expressions for different flow parameters such as flow rate, resistance to flow, wall shear stress and an ANN Model is designed for optimum solution of the problem considering the flow rate, resistance to flow, wall shear stress as output while radial viscosity parameter, fluid behavior index, yield stress, stenosis size and axis of artery as input. Computational model provides reasonable predictive performance which is close to the experimental values. The Levenberg–Marquardt Algorithm (LMA) was found the best of BP algorithms with a minimum mean squared error (MSE) for training and cross validation. In this paper we have concluded that the flow rate decreases while resistance to flow and wall shear stress increases with the increase in stenosis size and also the comparison of these parameters has been done for linear and quadratic variation of viscosity.
Keywords: Flow rate; Herschel- Bulkley fluid model; Overlapping stenosis; Resistance to flow; Variable viscosity; Wall shear stress; Levenberg–Marquardt Algorithm.

I. INTRODUCTION

In recent times, the investigations on blood flow modeling in different flow situations have gained momentum and created a lot of enthusiasm among the researchers. The quest for knowing the functioning of human physiological systems is pretty old and the motive behind this continued search is to achieve an insight to this complicated and complex systems. Mathematical modeling is the application of mathematics to explain and predict real world behavior. Scott- Blair et.al. [1] have reported that Herschel- Bulkley model is a better model than Casson’s model, they observed that the Casson fluid model can be used for moderate shear rates in smaller diameter tubes whereas the Herschel-Bulkley fluid model can be used at still lower shear rate flow in very narrow arteries where the yield stress is high. Chakravarthy et.al. [2] have reviewed the studied effects of overlapping stenosis on arterial flow problem analytically by assuming the pressure variation only along the axis of tube. Layek et.al. [3] have analyzed the theoretical investigation of overlapping stenosis on flow characteristics of blood considering the pressure variation in both the radial and axial directions of the arterial segment under consideration. Misra et.al. [4] have developed a Herschel-Bulkley fluid model and observed that the resistance to flow and skin friction increase as stenosis height increases. Arora et.al. [5] have discussed extensively that the velocity and flow rate increase with the increase of the peripheral layer thickness and decrease with the increase of the angle of tapering and depth of the stenosis. Neural network provides an approach to obtain accurate numerical values in a computationally less intensive fashion. Singh et.al. [6] have studied the effect of multiple radially non- symmetric stenosis with radial viscosity variation in an artery.

Arora et.al. [7] have designed the model for traffic noise system between Agra and Mathura highway using the Levenberg Marquardt Algorithm (LMA) for optimum network with a minimum Mean Square Error (MSE) for training and cross validation. Singh et.al. [8] have considered the effect of the plug flow on the flow characteristics of bile through diseased cystic duct: Casson Model analysis. Singh et.al. [9] have worked on the effect of radial magnetic field on flow of blood in a catheterized tapered artery with mild stenosis. Singh et.al. [10] have studied effect on the flow of bile due to radially varying viscosity in the cystic duct. Arora et.al. [11] have developed a mathematical model by treating blood as Non Newtonian fluid characterized by generalized power law model described the prediction of higher degree blockage of the stenosed artery with higher drop of blood pressure at the stenosis region considering single layer modeling technique. Singh et.al. [12] have considered the effect of radial viscosity variation on Non-Newtonian flow of blood in an overlapping stenosed artery. Singh et.al [13] have studied the effect of axial viscosity variation through an atherosclerotic artery: A Non-Newtonian fluid model. Singh et.al. [14] have investigated the analysis of the effect of the peripheral viscosity on bile flow characteristics through cystic duct with stone: Study of two-layer model with squeezing.

The present piece of work emphasises on comparative analysis of experimental and analytical study for the Non-Newtonian behavior of blood flow through overlapping stenosis: Computational Modeling Approach. In this paper, the flow of blood is assumed to be incompressible, fully developed, laminar and one dimensional.

II. FORMULATION OF PROBLEM

Consider the axisymmetric laminar flow of blood through a circular cylindrical tube under a constant pressure gradient with an overlapping constriction specified at the position as shown in Figure 1.

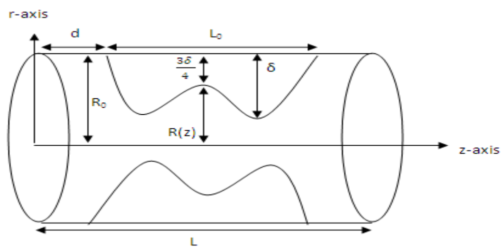

Figure 1 Geometry of arterial overlapping stenosis

The geometry of the stenosis which is assumed to be manifested in the arterial segment is described Chakravarty and Mandal [2] as

$$\frac{R(z)}{R_0} = 1 - \frac{3}{2}\frac{\delta}{R_0 L_0^4}\left[11(z-d)L_0^3 - 47(z-d)^2 L_0^2 + 72(z-d)^3 L_0 - 36(z-d)^4\right],\ d \le z \le d + L_0$$

$$= 1 \qquad \text{otherwise} \qquad (1)$$

where R(z) and R_0 are the radius of the artery with and without stenosis, respectively, L_0 is the length of the stenosis and d indicates its location, δ is the maximum height of the stenosis into the lumen, appears at two locations: $z = d + \frac{1}{6}L_0$ and $z = d + \frac{5}{6}L_0$ .The stenosis height at $z = d + \frac{1}{2}L_0$ from origin, called critical height, is $\frac{3\delta}{4}$ .The constitutive equations for Herschel- Bulkley fluid is [4]:

$$\left(-\frac{\partial u}{\partial r}\right) = \frac{1}{\mu(r)}(\tau - \tau_0)^n \qquad , \tau \ge \tau_0$$
$$= 0 \qquad , \tau < \tau_0 \qquad (2a)$$

where u stands for the axial velocity of blood, τ_0 is the yield stress, $\mu(r)$ is the fluid viscosity and n is fluid behavior index.

Viscosity $\mu(r)$ is given by,

$$\mu(r) = \mu_0\left[1 - k\left(\frac{r}{R_0}\right)^{\beta}\right] \qquad (2b)$$

where μ_0 is the viscosity of plasma, k is radial viscosity parameter and β is a parameter which shows the viscosity variation .

The boundary conditions are

$$\frac{\partial u}{\partial r} = 0 \text{ at } r = 0 \qquad (3a)$$

$$u = 0 \text{ at } r = R(z) \qquad (3b)$$

τ is finite at $r = 0$ (3c)

$P = P_0$ at $z = 0$ and $P = P_L$ at $z = L$ (3d)

The Navier -Stokes equation in cylindrical coordinate system for blood is given by [4]:

$$0 = -\frac{\partial p}{\partial z} - \frac{1}{r}\frac{\partial(r\tau)}{\partial r} \qquad (4a)$$

$$0 = \frac{\partial p}{\partial r} \qquad (4b)$$

III. MATERIALS AND METHODS

a) Computational Model (ANN approach)

The proposed ANN model is divided into three layers of nodes (neurons): input, hidden and output. The linkages between input and hidden nodes and hidden and output nodes are associated by synapses. Weighted connections are established between the input and output data relationship. Radial viscosity parameter, fluid behavior index, yield stress, stenosis size and axis of artery are taken as input while flow rate, flow resistance, wall shear stress as output. For optimum network, different architecture was tested in which each pattern consisting of an input and output vector is normalized to avoid numerical error propagation during the learning process. The data set is normalized to values lying between 0.01 to 0.99. The sum of the modified signals is performed through the Sigmoid axon transfer activation function.

Using supervised learning based on evolutionary search, an ANN can learn the mapping from one data space to another data space. The quadratic shape of the error computed at the output side by comparing the measured**, *exp vi*** and the simulated**, *simp vi*** values of blood flow velocities is obtained. Error is propagated backward from the output layer, to the hidden layer and finally to the input layer. Each generation of the process in back propagation constitutes two sweeps: forward activation to produce a solution, and a backward propagation of the computed error to modify the weights of the synapses and the biases vector on hidden or output layers. The algorithm provides the weight and bias adjustments in the backward sweep corresponding to the optimal configuration of the ANN. This is a nonlinear procedure because of the nonlinear threshold element contained in each node, and its behavior is very complex due to the layered structure. However this nonlinear behavior enables to analyze the changes in blood flow pattern bifurcation neighborhood associated with arterial overlapping stenosis or occlusion diseases.

b) Design of Experiment

Fluid Preparation and Rheology

Newtonian salt-water solution is taken as Fluid which is then mixed with carbopol which is widely used as thickener, stabilizer and suspending agent. The rheology of the Non-Newtonian fluid thus formed is controlled by the concentration and PH of the solution and it is then mixed with water which makes an acidic solution with no yield stress. In our experiments the concentration of the fluid was not very high and its rotating speed is usually set between 100 to 400 rpm.

	Carbopol %	NaOH %	Tu	N
Fluid	0.1025	0.0292	1.15	2/3= 0.666

Table 1 Composition and properties of the displaced fluid used in our experiments

A rheological model that fits with the shear behavior of carbopol behavior is the Herschel– Bulkley model.

$$\left(-\frac{\partial u}{\partial r}\right) = \frac{1}{\mu(r)}\,(\tau-\tau_0)^n \qquad ,\ \tau \geq \tau_0$$
$$= 0 \qquad ,\ \tau < \tau_0$$

Figure 2 Design of experiment in Bio-Medical lab

IV. SOLUTION OF PROBLEM

Substituting the value of τ from equation (2a) in the equation (4a), we get

$$0 = -\frac{\partial p}{\partial z} - \frac{1}{r}\frac{\partial}{\partial r}\left[r\left\{ \mu(r)^{\frac{1}{n}}\left(-\frac{\partial u}{\partial r}\right)^{\frac{1}{n}} + \tau_0 \right\}\right] \qquad (5)$$

Integrating equation (5) with respect to r, we get, $\left(-\frac{\partial u}{\partial r}\right)^{\frac{1}{n}} = \frac{1}{\mu(r)^{\frac{1}{n}}}\left[\frac{r}{2}\left(-\frac{\partial p}{\partial z}\right) - \tau_0\right]$ (6)

The constant flux is given by, $Q = \int_0^{R(z)} 2\pi r u\, dr$

$$= \int_0^{R(z)} \pi r^2\left(-\frac{\partial u}{\partial r}\right)dr = \left(-\frac{1}{2}\frac{\partial p}{\partial z}\right)^n \pi\, I(R(z)) \qquad (7)$$

where $I(R(z)) = \int_0^{R(z)} \frac{r^2}{\mu(r)}\left[r + \frac{2\tau_0}{\left(\frac{\partial p}{\partial z}\right)}\right]^n dr$

From (7), $\left(-\frac{\partial p}{\partial z}\right) = 2\left(\frac{Q}{\pi\, I(R(z))}\right)^{\frac{1}{n}}$ (8)

For solving pressure drop ∇P, integrating (8) and using boundary condition (3d),

$$\nabla P = -(P_L - P_0) = 2\left(\frac{Q}{\pi}\right)^{\frac{1}{n}}\int_0^L \frac{1}{\left(I(R(z))\right)^{\frac{1}{n}}}\,dz \qquad (9)$$

Resistance to flow is given by,

$$\lambda = \frac{\nabla P}{Q} = 2\left(\frac{Q^{1-n}}{\pi}\right)^{\frac{1}{n}}\int_0^L \frac{1}{\left(I(R(z))\right)^{\frac{1}{n}}}\,dz \qquad (10)$$

Wall shear stress at the wall is given by,

$$\tau = \tau_0 + \left[-\frac{\partial u}{\partial r}\mu(r)\right]^{\frac{1}{n}} = R(z)\left(\frac{Q}{\pi\, I(R(z))}\right)^{\frac{1}{n}} \qquad (11)$$

For solving this problem, we consider two cases of viscosity variation:

Case I: Consider $\beta = 1$ in equation (2b) i.e., linear variation of viscosity, then

$$\mu(r) = \mu_0\left[1 - k\left(\frac{r}{R_0}\right)\right]$$

Then flow rate is given by, $Q = \left(-\frac{1}{2}\frac{\partial p}{\partial z}\right)^n \frac{\pi}{\mu_0} I_1(R(z)$ (12)

where $I_1(R(z)) = \int_0^{R(z)}\left[r^2\left(r + \frac{2\tau_0}{\left(\frac{\partial p}{\partial z}\right)}\right)^n + \frac{kr^3}{R_0}\left(r + \frac{2\tau_0}{\left(\frac{\partial p}{\partial z}\right)}\right)^n\right]dr$

Resistance to flow is given by,

$$\lambda = \frac{\nabla P}{Q} = 2\left(\frac{\mu_0\, Q^{1-n}}{\pi}\right)^{\frac{1}{n}}\int_0^L \frac{1}{\left(I_1(R(z))\right)^{\frac{1}{n}}}\,dz$$

$$= 2\left(\frac{\mu_0\, Q^{1-n}}{\pi}\right)^{\frac{1}{n}}\left[\int_0^d \frac{1}{\left(I_1(R(z))\right)^{\frac{1}{n}}}\,dz + \int_d^{d+L_0} \frac{1}{\left(I_1(R(z))\right)^{\frac{1}{n}}}\,dz + \int_{d+L_0}^{L} \frac{1}{\left(I_1(R(z))\right)^{\frac{1}{n}}}\,dz\right]$$

At $R(z) = R_0$, Resistance to flow is given by,

$$\lambda_w = 2\left(\frac{\mu_0\, Q^{1-n}}{\pi}\right)^{\frac{1}{n}}\int_0^L \frac{1}{\left(I_0(R_0)\right)^{\frac{1}{n}}}\,dz$$

where, $I_0(R_0) = \int_0^{R_0}\left[r^2\left(r + \frac{2\tau_0}{\left(\frac{\partial p}{\partial z}\right)}\right)^n + \frac{kr^3}{R_0}\left(r + \frac{2\tau_o}{\left(\frac{\partial p}{\partial z}\right)}\right)^n\right]dr$

$$\text{Then } \lambda^- = \frac{\lambda}{\lambda_w} = \frac{\int_0^L \frac{1}{\left(I_1(R(z))\right)^{\frac{1}{n}}}dz}{\int_0^L \frac{1}{\left(I_0(R_0)\right)^{\frac{1}{n}}}dz} \qquad (13)$$

Wall shear stress for linear viscosity variation is given by,

$$\tau = R(z)\left(\frac{Q\mu_0}{\pi\, I_1(R(z))}\right)^{\frac{1}{n}}$$

At $R(z) = R_0$, i.e. in absence of stenosis, the wall shear stress is given by,

$$\tau_w = R_0\left(\frac{Q\mu_0}{\pi\, I_0(R_0)}\right)^{\frac{1}{n}}$$

$$\text{Then } \tau^- = \frac{\tau}{\tau_w} = \frac{R(z)}{R_0}\left(\frac{I_0(R_0)}{I_1(R(z))}\right)^{\frac{1}{n}} \qquad (14)$$

Case II: Consider $\beta = 2$ in equation (2b), i.e., quadratic variation of viscosity, then

$$\mu(r) = \mu_0\left[1 - k\left(\frac{r}{R_0}\right)^2\right]$$

Then flow rate is given by, $Q = \left(-\frac{1}{2}\frac{\partial p}{\partial z}\right)^n \frac{\pi}{\mu_0} I_2(R(z))$ (15)

where $I_2(R(z)) = \int_0^{R(z)}\left[r^2\left(r + \frac{2\tau_0}{\left(\frac{\partial p}{\partial z}\right)}\right)^n + \frac{kr^4}{R_0^2}\left(r + \frac{2\tau_0}{\left(\frac{\partial p}{\partial z}\right)}\right)^n\right]dr$

Resistance to flow is given by,

$$\lambda = \frac{\nabla P}{Q} = 2\left(\frac{\mu_0 Q^{1-n}}{\pi}\right)^{\frac{1}{n}} \int_0^L \frac{1}{\left(I_2(R(z))\right)^{\frac{1}{n}}}\, dz$$

$$= 2\left(\frac{\mu_0 Q^{1-n}}{\pi}\right)^{\frac{1}{n}} \left[\int_0^d \frac{1}{\left(I_2(R(z))\right)^{\frac{1}{n}}}\, dz + \int_d^{d+L_o} \frac{1}{\left(I_2(R(z))\right)^{\frac{1}{n}}}\, dz + \int_{d+L_0}^L \frac{1}{\left(I_2(R(z))\right)^{\frac{1}{n}}}\, dz\right]$$

At $R(z) = R_0$, Resistance to flow is given by,

$$\lambda_w = 2\left(\frac{\mu_0 Q^{1-n}}{\pi}\right)^{\frac{1}{n}} \int_0^L \frac{1}{\left(I_0(R_0)\right)^{\frac{1}{n}}}\, dz$$

where $I_0(R_0) = \int_0^{R_0}\left[r^2\left(r + \frac{2\tau_0}{\left(\frac{\partial p}{\partial z}\right)}\right)^n + \frac{kr^4}{R_0^2}\left(r + \frac{2\tau_0}{\left(\frac{\partial p}{\partial z}\right)}\right)^n\right] dr$

$$\text{Then}\lambda^- = \frac{\lambda}{\lambda_w} = \frac{\int_0^L \frac{1}{\left(I_2(R(z))\right)^{\frac{1}{n}}}\, dz}{\int_0^L \frac{1}{\left(I_0(R_0)\right)^{\frac{1}{n}}}\, dz} \qquad (16)$$

Wall shear stress for quadratic viscosity variation is given by, $\tau = R(z)\left(\frac{Q\mu_0}{\pi\, I_2(R(z))}\right)^{\frac{1}{n}}$

$$\text{Then } \tau^- = \frac{\tau}{\tau_w} = \frac{R(z)}{R_0}\left(\frac{I_0(R_0)}{I_2(R(z))}\right)^{\frac{1}{n}} \qquad (17)$$

V. RESULTS AND DISCUSSON

The present paper focuses that the influence of the radial variation of viscosity in case of overlapping stenosis on various flow parameters such as flow rate, resistance to flow and wall shear stress. The analytical expressions for flow parameters are given by equations from 12 to 17 and the computational results are obtained from the present study and are shown graphically.

Neural Network Toolbox Neuro Solution 6.0 ® mathematical software was used. A single-layer ANN model was designed considering radial viscosity parameter, fluid behavior index, yield stress, stenosis size and axis of artery as input while the flow rate, flow resistance, wall shear stress as output with sigmoid axon transfer function. Network represents functional relationship between inputs and output, provided sigmoid layer has enough neurons. Levenberg- Marquardt Algorithm is fastest training algorithm for network of moderate size, therefore, used in the present study.

The sigmoid axon was considered transfer function with 0.7 momentums. Series of experiment resulted into the evaluation of performance based on 65 % data for training, 15 % data for testing and 20 % data for cross validation at 1500 Epoch with 0.70000 momentums. The performance of network simulation was evaluated in terms of mean square error (MSE) criterion. The minimum MSE in the group of six variables was determined for training and cross validation are 0.001658332 and 0.001658332 respectively.

Best Networks		Cross Validation
Epoch	1500	1491
Minimum MSE	0.001658332	0.007990010
Final MSE	0.001658332	0.0078251210

Table 2 shows the result obtained by the Neural Network simulation for training, cross validation and testing data sets

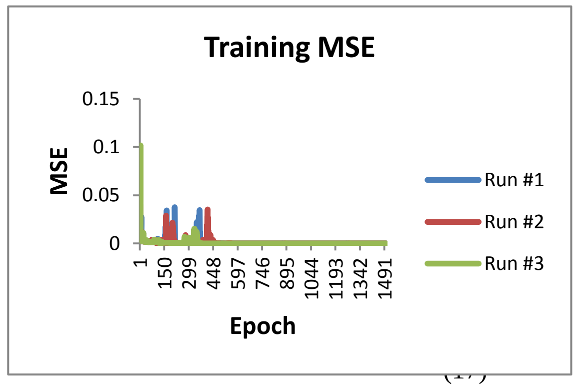

Figure 3(a) Software value of MSE for training

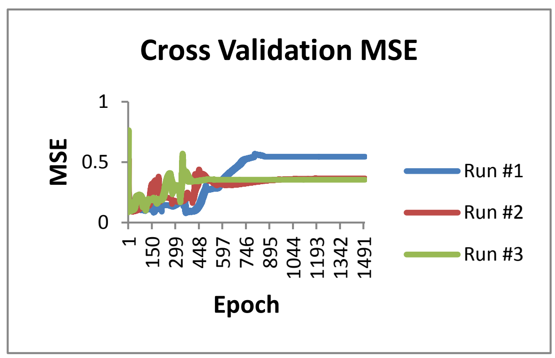

Figure 3(b) Software value of MSE for testing

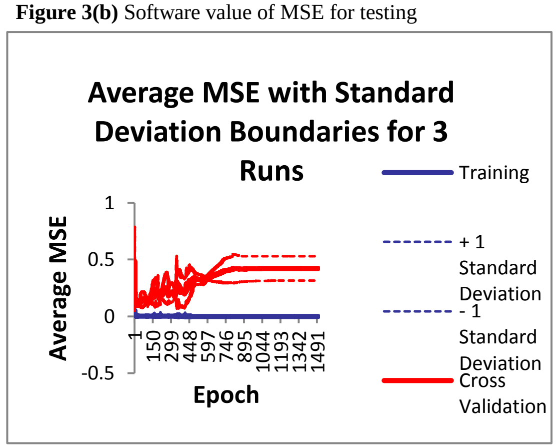

Figure 3(c) Software value of MSE for cross validation

Testing and Sensitivity analysis

Sensitivity analysis is a powerful and illuminating methodology. Sensitivity index has computed by Haffman and Gardener, 1983as

S.I. = (D_{max} –D_{min}) / D_{max}

where D_{max} is the output result and D_{min} is the result for minimum value of the parameters.

A sensitivity analysis was conducted to determine the degree of effectiveness of variables. Performance of the group of input vectors included radial viscosity parameter, fluid behavior index, yield stress, stenosis size and axis of artery.

The developed network model was examined for its ability to predict the response of experimental data not forming the part of the training program. The network was finally tested for training, cross validation and testing data sets. The comparison results of desired outputs and network output are shown in Figure 4(b).

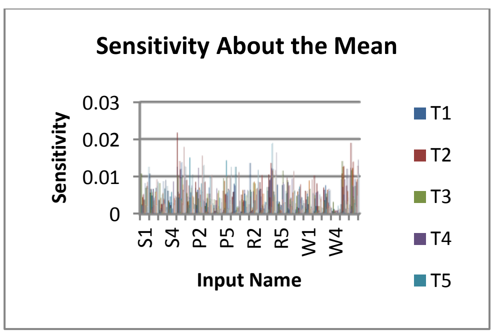

Figure 4(a) Sensitivity analysis

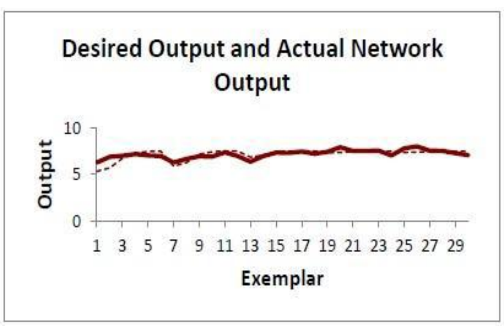

Figure 4(b) Desired output and actual output

VI. CONCLUSION

The present piece of work that the influence of the radial variation of viscosity in case of overlapping stenosis on various flow parameters such as flow rate, resistance to flow and wall shear stress. The analytical expressions for flow parameters are given by equations from 12 to 17 and the computational results are obtained from the present study and are shown graphically.

Figure 5(a) shows that with increase in the value of radial viscosity parameter *k*, the flow rate increases as increase in the value of *k* leads to the reduction of viscosity.

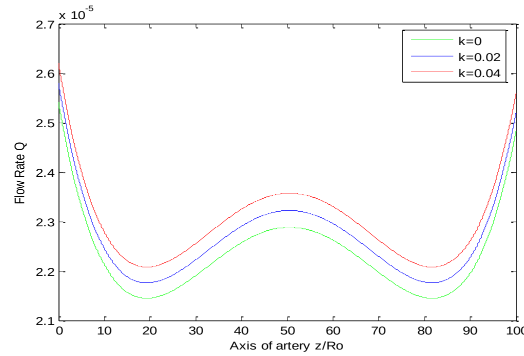

Figure 5(a) Graph of Flow Rate versus Axis of Artery varying radial viscosity parameter *k*

Figure 5(b) and 5(c) shows that flow rate increases as the value of fluid behavior index *n* and yield stress τ_0 increases.

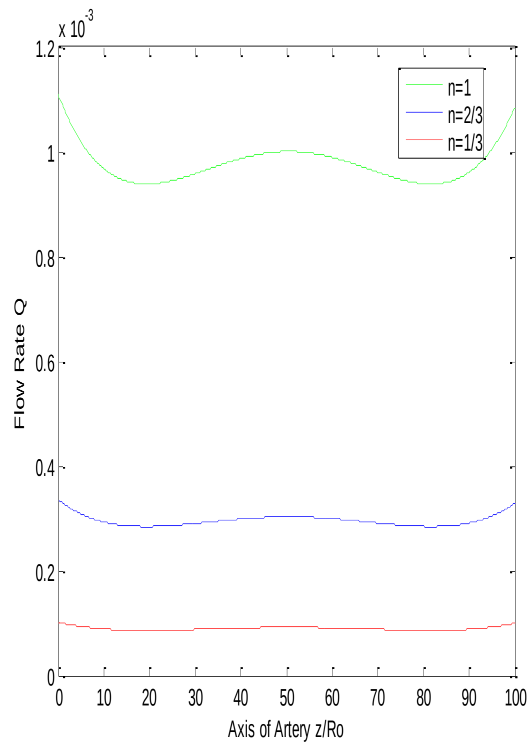

Figure 5(b) Graph of Flow Rate versus Axis of Artery varying fluid behavior index *n*

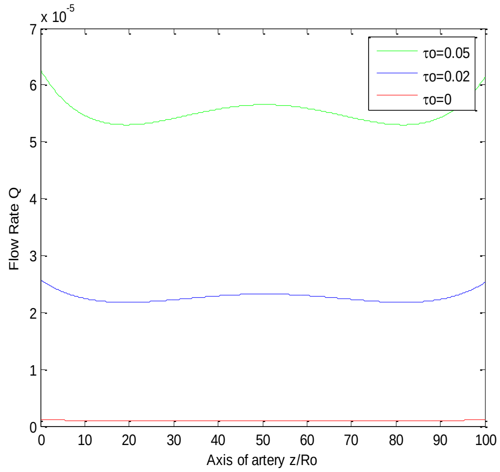

Figure 5(c) Graph of Flow Rate versus Axis of Artery varying yield stress τ_0

In Figure 6, the comparison has been done between linear and quadratic variation of viscosity for flow rate and have observed that flow rate for linear variation of viscosity is more as compared to quadratic variation of viscosity.

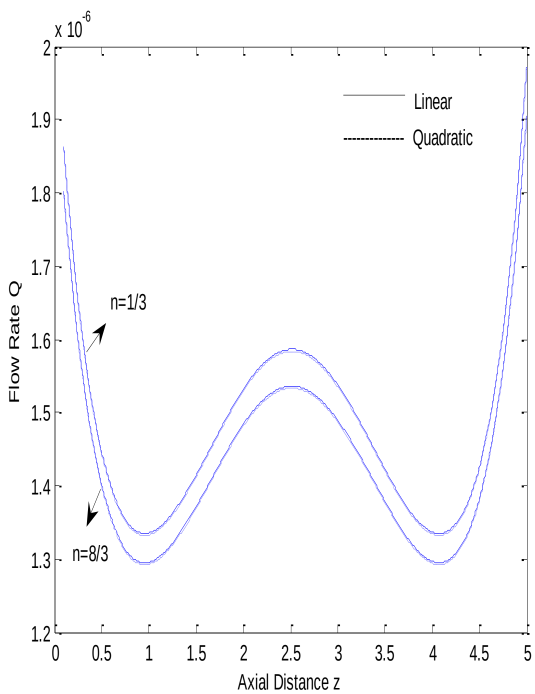

Figure 6 Comparison of Flow Rate for Linear and Quadratic Variation of Viscosity varying fluid behavior index n

In Figures 7(a)-7(c), it has been noticed that flow rate decreases as the stenosis size increases and with the increase in the value of radial viscosity parameter k, fluid behavior index n and yield stress τ_0, the flow rate also increases.

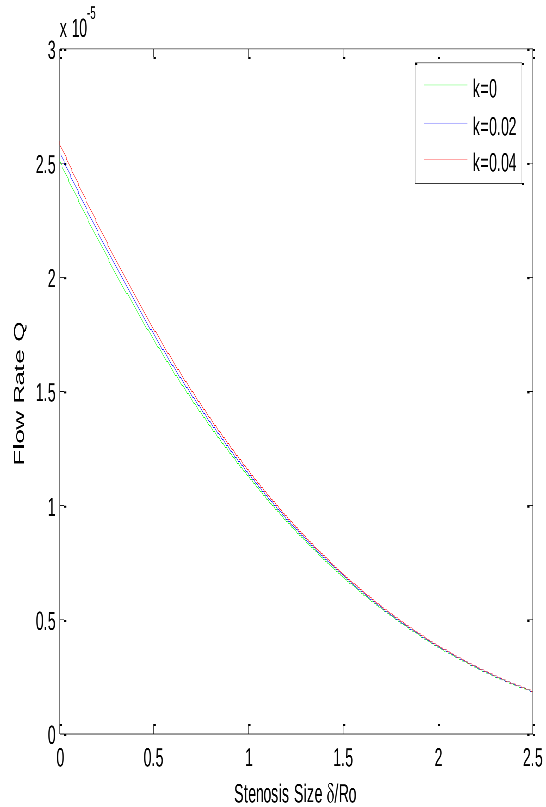

Figure 7(a) Graph of Flow Rate versus Stenosis Size varying radial viscosity parameter k

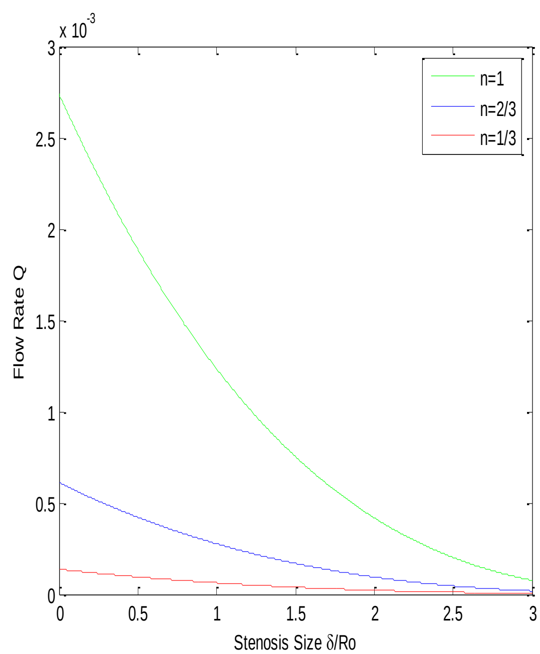

Figure 7(b) Graph of Flow Rate versus Stenosis Size varying fluid behavior index n

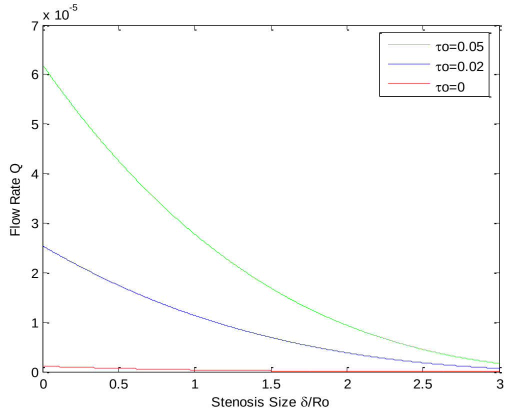

Figure 7(c) Graph of Flow Rate versus Stenosis Size varying yield stress τ_0

Figure 8(a)-8(c) shows the variation of flow resistance with the stenosis size varying *k, n*, τ_0 respectively. It shows that with the increase in the stenosis size, flow resistance also increases and particularly in figures 8(a) and 8(b), with increase in the value of radial viscosity parameter *k* and fluid behavior index *n*, flow resistance increases.

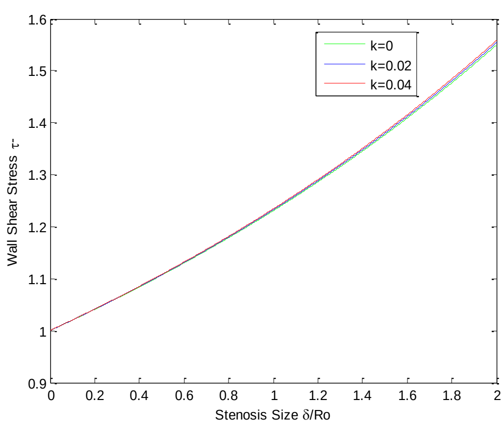

Figure 8(a) Graph of Wall Shear Stress versus Stenosis Size varying radial viscosity parameter *k*

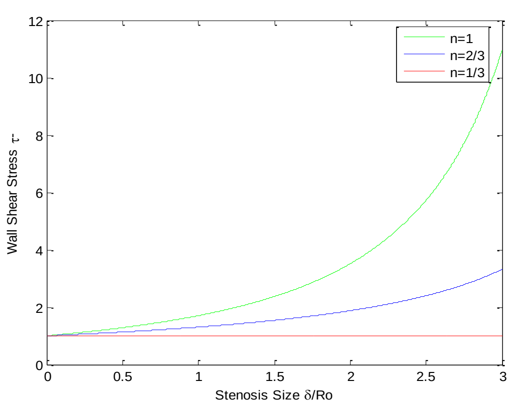

Figure 8(b) Graph of Wall Shear Stress versus Stenosis Size varying fluid behavior index *n*

Figure 8(c) shows that as the yield stress τ_0 increases, the wall shear stress decreases.

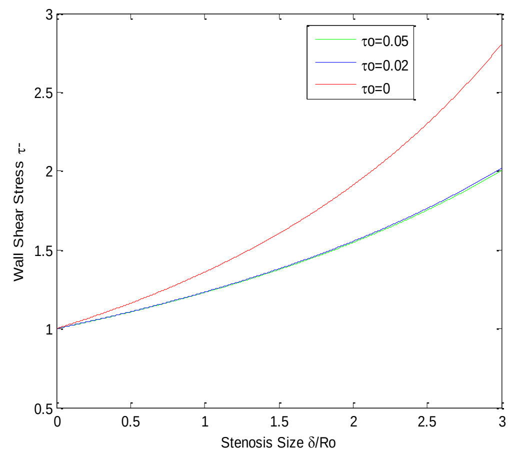

Figure 8(c) Graph of Wall Shear Stress versus Stenosis Size varying yield stress τ_0

Figure 9 shows that stress increases as we move along the axis of the artery and becomes maximum at the maximum height δ of the stenosis and then start to decrease and attains a value at stenosis height $\frac{3\delta}{4}$ and then again increases and reaches the maximum value and then starts to decrease as the stenosis height reduces. For the particular values of *k*, wall shear stress in case of quadratic variation of viscosity is more in comparison to linear variation of viscosity.

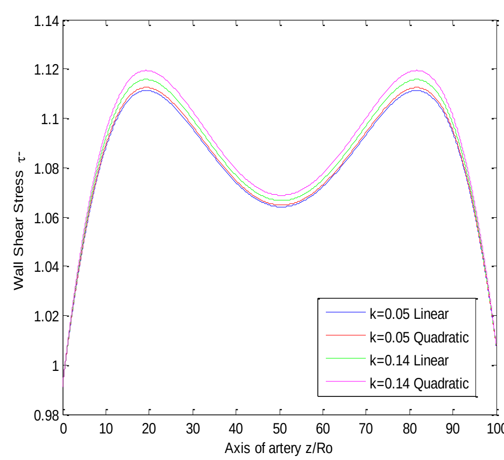

Figure 9 Comparison of Wall Shear Stress for Linear and Quadratic Variation of Viscosity varying radial viscosity parameter *k*

Thus, the results demonstrate that the present model is capable of predicting the hemodynamic features most interesting to physiologists and, thus, it is concluded that the presence of the peripheral layer helps in the functioning of the diseased arterial system. The developed ANN model could describe the behavior of the complex interaction process within the range of experimental conditions adopted.

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