Free Convective Heat And Mass Transfer Induced By A Constant Mass Flux On A Parabolic Started Vertical Plate With Variable Temperature

E.Geetha[1*], R. Muthucumaraswamy[2]

[1]Department of Mathematics, Sri Chandrasekharendra Saraswathi Viswa Mahavidyalaya University, Enathur, Kanchipuram 631561, India

[2] Department of Applied Mathematics, Sri Venkateswara College of Engineering, Sriperumbudur 602 105, India.

****Corresponding author: geethamuthu06@gmail.com***

Abstract-This work presents the effects on a parabolic started vertical plate with variable temperature which is induced by a uniform mass flux. Here the Bousseinq's equations for the momentum, energy and mass equations are considered which is transformed into linear partial differential equations by using non-dimensional quantities. The governing equations are solved by the laplace transform technique. The concentration, temperature and velocity profiles are analyzed graphically for the different parameters like thermal Grashof number, mass Grashof number, Schmidt number, Prandtl number and time. It is observed that velocity enhances for increasing values of time or thermal Grashof number or mass Grashof number. The shear stress for different values of thermal Grashof number, mass Grashof number, Schmidt number, Prandtl number and time are obtained and it is noticed that when t enhances the values of shear stress decrease and when the mass Grashof number increases the value of skin friction decrease.

Key words: Parabolic, vertical plate, constant mass flux, variable temperature, Skin friction.

I.INTRODUCTION

The unsteady natural convection flow of a viscous fluid along a vertical plate is a classical problem in fluid mechanics and heat transfer with significance for a variety of engineering applications. The study of convective flow, heat and mass transfer has been an active field of research, as it plays a crucial role in diverse applications such as cooling of electronic devices by fans, cooling of nuclear reactors during emergency shutdown, solar central receivers, exposed to wind current and chemical catalytic reactions.

Transient free convection from a vertical plate was analyzed by Siegel[1]. Joshi et al[2] studied the transition of vertical natural convection flows in water. Patterson et al[3] discussed the boundary layer development on a semi-infinite suddenly heated vertical plate. Das et al[4] analyzed the transient free convection flow past an infinite vertical plate with periodic temperature variation. Unsteady natural convection in the vicinity of a doubly- infinite vertical plate was discussed by Schetz et al[5]. Illingworth[6] have studied the unsteady laminar flow of a gas near an infinite flat plate. Laminar free convection in a vertical slot has studied by Elder[7].Goldstein[8] has studied the transient free convection about vertical plates and circular cylinders. Prandtl number dependence of unsteady natural convection along a vertical plate in a stably stratified fluid has discussed by Alan Shapiro et al[9]. Abbas[10] has analyzed the effects of magnetohydrodynamic flow past a vertical plate with variable surface temperature. The mass transfer effects on MHD viscous flow past an impulsively started infinite vertical plate with constant mass flux was studied by Saravana et al[11]. Soundalgekar et al[12] has analyzed the mass transfer effects on flow past an impulsively started infinite vertical plate with constant mass flux. Bejan and Khair[13] have discussed the heat and mass transfer by natural convection in a porous medium. Unsteady convective boundary layer flow of a viscous fluid at a vertical surface with variable fluid properties was analyzed by Vajravelu et al[14]. Elbashbeshy[15]has studied heat and mass transfer along a vertical plate with variable surface temperature and concentration in the presence of magnetic field. Ganesan and Rani [16]has studied the unsteady free convective on vertical cylinder with variable heat and mass flux. Gokhale and Sammanhave[17] discussed the effects of mass transfer on the transient free convective flow of a dissipative fluid along a semi-infinite vertical plate with constant heat flux. Theoretical considerations of combined thermal and mass transfer from vertical flat plate were studied by Somers[18].Takhar et al [19]studied the transient free convection past a semi-infinite vertical plate with variable surface temperature. The effects of mass transfer and heat sources on the flow past an accelerated infinite vertical plate was analyzed by Soundalgekar et al[20]. Das et al[21] has studied the exact solution of mass transfer effects on flow past an impulsively started infinite vertical plate with constant mass flux. Radiation and mass transfer effects on moving vertical plate with variable temperature and viscous dissipation was investigated by Satyanarayana et al[22]. Abd el-nabyet al[23] have studied the effects of radiation on unsteady free convective flow past a semi-infinite vertical plate with variable surface temperature using Crank-Nicolson finite difference method. Laplace Technique on Magneto hydrodynamic radiating and chemically reacting fluid over an infinite vertical surface was discussed by Sahin Ahmed et al[24]. Vijaya et al[25] has studied the MHD free convective flow past an exponentially accelerated vertical plate with variable temperature and variable mass diffusion. Here the effect of natural convective heat and mass transfer of an incompressible fluid flow past on a parabolic started vertical plate with variable temperature and uniform mass flux has not been studied in the literature.

The objective of this paper is the effect of a laminar free convective flow of an incompressible fluid past on a parabolic started vertical plate with variable temperature which is induced by constant mass flux. The governing equations are solved by laplace transform technique.

II.MATHEMATICAL FORMULATION

Here the unsteady flow of a viscous incompressible fluid past a parabolic started infinite vertical plate with variable temperature and uniform mass flux has been considered. The fluid considered here is gray, absorbing-emitting radiation but a non-scattering medium. The x-axis is taken along the plate in the vertical upward direction and the y-axis is taken normal to the plate. Initially it is assumed that the plate and fluid are at the same temperature T_∞ in the stationary condition with concentration level C'_∞ at all the points. At time t' > 0, the plate is given a parabolic motion in its own plane with velocity $u = u_0 t'^2$. At the same time the plate temperature is raised linearly with time t' and also mass flux is carried out uniformly from the plate. Then by usual Boussinesq's approximation, the unsteady flow is governed by the following set of equations.

$$\frac{\partial u}{\partial t'} = g\beta(T - T_\infty) + g\beta^*(C' - C'_\infty) + \nu\frac{\partial^2 u}{\partial y^2} \quad (1)$$

$$\rho C_p \frac{\partial T}{\partial t'} = k\frac{\partial^2 T}{\partial y^2} \quad (2)$$

$$\frac{\partial C'}{\partial t'} = D\frac{\partial^2 C'}{\partial y^2} \quad (3)$$

With the following initial and boundary conditions

$u = 0,\ T = T_\infty,\ C' = C'_\infty,$ for all y, t' ≤ 0.

$$t' > 0, u = u_0 t'^2, T = T_\infty + (T_w - T_\infty)At', \frac{\partial C'}{\partial y} = -\frac{j''}{D}$$

at y = 0 (4)

$u \to 0, T \to T_\infty, C' \to C'_\infty$ as $y \to \infty$

On introducing the following non – dimensional quantities:

$$U = u\left(\frac{u_0}{\nu^2}\right)^{1/3}, t = \left(\frac{u_0^2}{\nu}\right)^{1/3} t', Y = y\left(\frac{u_0}{\nu^2}\right)^{1/3},$$

$$\theta = \frac{T - T_\infty}{T_W - T_\infty}, C = \frac{C' - C'_\infty}{\dfrac{j''\nu^{2/3}}{Du_0^{1/3}}}, Gr = \frac{g\beta(T - T_\infty)}{(\nu u_0)^{1/3}},$$

$$Gc = \frac{g\beta^*\left(\dfrac{j''\nu^{2/3}}{Du_0^{1/3}}\right)}{(\nu u_0)^{1/3}}, \Pr = \frac{\mu C_p}{k}, Sc = \frac{\nu}{D},$$

$$A = \left(\frac{u_0^2}{\nu}\right)^{1/3} \quad (5)$$

In equations (1) to (4) leads to

$$\frac{\partial U}{\partial t} = Gr\theta + GcC + \frac{\partial^2 U}{\partial Y^2} \quad (6)$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{\Pr}\frac{\partial^2 \theta}{\partial Y^2} \quad (7)$$

$$\frac{\partial C}{\partial t} = \frac{1}{Sc}\frac{\partial^2 C}{\partial Y^2} \quad (8)$$

The initial and boundary conditions in non dimensional quantities are

$U = 0, \theta = 0, C = 0$ for all Y , t ≤ 0

$$t > 0, U = t^2, \theta = t, \frac{\partial C}{\partial Y} = -1 \text{ at } Y = 0 \quad (9)$$

$U \to 0, \theta \to 0, C \to 0$ as $Y \to \infty$ (10)

III.METHODS OF SOLUTIONS

The dimensionless governing equations (6) to (8) and the corresponding initial and boundary conditions (9) are tackled using Laplace transform technique.

The unsteady linear coupled partial differential equations of a viscous flow have been solved by using laplace transform technique and the solutions are obtained in terms of exponential and complementary error functions.

$$C = 2\sqrt{t}\left[\frac{\exp(-\eta^2 Sc)}{\sqrt{\pi}\sqrt{Sc}} - \eta\, erfc(\eta\sqrt{Sc})\right]$$

$$\theta = t\left[(1 + 2\eta^2 \Pr)erfc(\eta\sqrt{\Pr}) - \frac{2\eta\sqrt{\Pr}}{\sqrt{\pi}}\exp(-\eta^2 \Pr)\right]$$

(11)

$$U=\frac{t^2}{3}\begin{bmatrix}(3+12\eta^2+4\eta^4)erfc(\eta)-\\ \frac{\eta}{\sqrt{\pi}}(10+4\eta^2)\exp(-\eta^2)\end{bmatrix}$$

$$-at^2\begin{bmatrix}(3+12\eta^2+4\eta^4)erfc(\eta)-\\ (3+12\eta^2\,\mathrm{Pr}+4\eta^4(\mathrm{Pr})^2)erfc(\eta\sqrt{\mathrm{Pr}})\\ -\frac{\eta}{\sqrt{\pi}}(10+4\eta^2)\exp(-\eta^2)+\\ \frac{\eta\sqrt{\mathrm{Pr}}}{\sqrt{\pi}}(10+4\eta^2\,\mathrm{Pr})\exp(-\eta^2\,\mathrm{Pr})\end{bmatrix}$$

$$-bt\sqrt{t}\begin{bmatrix}\frac{4}{\sqrt{\pi}}(1+\eta^2)\exp(-\eta^2)-\\ \eta(6+4\eta^2)erfc(\eta)\\ -\frac{4}{\sqrt{\pi}}(1+\eta^2 Sc)\exp(-\eta^2 Sc)+\\ \eta\sqrt{Sc}(6+4\eta^2 Sc)erfc(\eta\sqrt{Sc})\end{bmatrix} \tag{12}$$

Where $a=\frac{Gr}{6(1-\mathrm{Pr})},b=\frac{Gc}{3\sqrt{Sc}(1-Sc)}$

Skin friction

The boundary layer produces a drag force on the plate due to the viscous stresses which are developed at the wall. The viscous stress at the surface of the plate is given by $\tau=-\left(\frac{\partial u(y,t)}{\partial y}\right)_{y=0}$

$$\tau=\frac{-1}{2\sqrt{t}}\begin{bmatrix}\frac{t^2}{3}\left(\frac{-16}{\sqrt{\pi}}\right)\\ -\frac{Grt^2}{6(1-\mathrm{Pr})}\left(\frac{-16}{\sqrt{\pi}}\right)\left(1-\sqrt{\mathrm{Pr}}\right)\\ -\frac{Gct\sqrt{t}}{3\sqrt{Sc}(1-Sc)}\left(-6+6\sqrt{Sc}\right)\end{bmatrix} \tag{13}$$

IV.RESULTS AND DISCUSSION

To gain a perspective of the physics of the flow regime, we have evaluated the effects of thermal Grashof number Gr, mass Grashof number Gc, Prandtl number Pr, Schmidt number Sc, and time t on the velocity, temperature, concentration profiles and skin friction. Here we consider

Gr = Gc = 5 >0 (cooling of the plate) ie., free convection current convey heat away from the plate into the boundary layer, t=0.2, Pr=0.71(air), Sc = 0.16 throughout the discussion.

Figure 1 displays the effects of time t (t = 0.2, 0.4, 0.6) on the concentration profile for Sc=0.16. It is observed that the concentration of the fluid enhances with the increasing values of t.

In Figure 2 we have presented the influence of Schmidt number Sc (Sc = 0.16, 0.6, 2.01) on the concentration profile at t=0.2. As the Schmidt number increases, the concentration decreases. This causes the concentration buoyancy effects to decrease yielding a reduction in the fluid velocity. Reductions in the concentration distributions are accompanied by simultaneous reductions in the velocity and concentration boundary layers.

Figure 3 shows the behavior of the temperature for different values of Prandtl number Pr(Pr =0.71, 2.0, 7.0) at t = 0.2. It is observed that an increase in the Prandtl number results a decrease

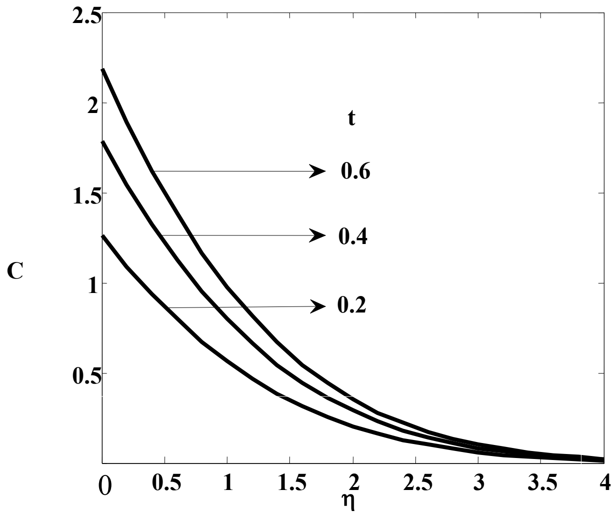

Figure.1 Concentration profiles for different values of t

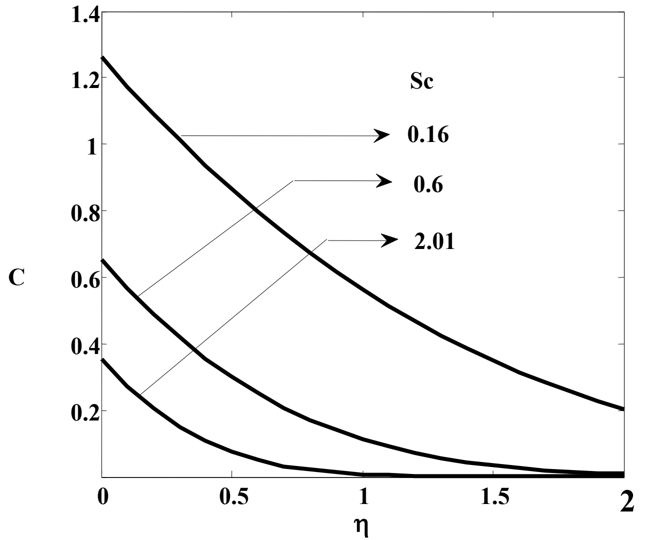

Figure.2. Concentration profiles for different values of Sc

of the thermal boundary layer thickness and in general lower average temperature within the boundary layer. The reason is that smaller values of Pr are equivalent to increase in the thermal conductivity of the fluid and therefore, heat is able to diffuse away from the heated surface more rapidly for higher values of Pr.

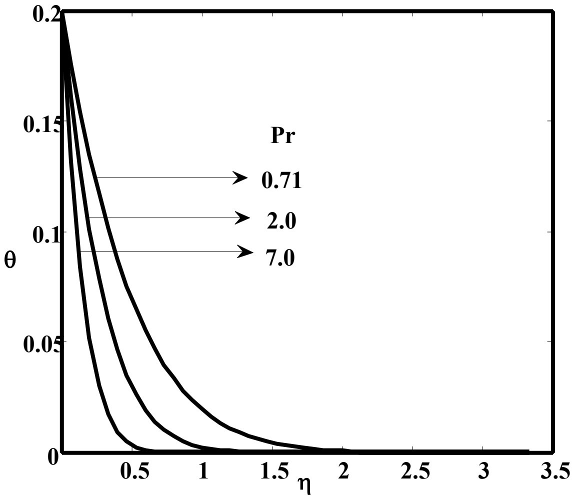

Figure.3. Temperature profiles for different values of Pr

Hence in the case of smaller Prandtl number as the thermal boundary layer is thicker and the rate of heat transfer is reduced.

For different values of time t (t =0.2, 0.4, 0.6) on the temperature profile are shown in Figure 4. It is noticed that an increase in time t results an increase in the temperature field.

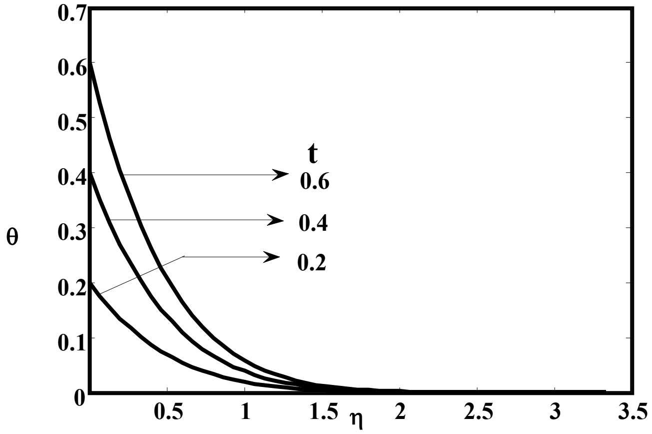

Figure.4. Temperature profiles for different values of t

Figure 5 displays the influence of the Prandtl number Pr(Pr = 0.71, 2.0, 7.0) on the velocity profiles at t=0.2. It is seen that velocity decrease as Pr increases. This is an agreement with the physical fact that the thermal boundary layer thickness decreases with increasing Pr.

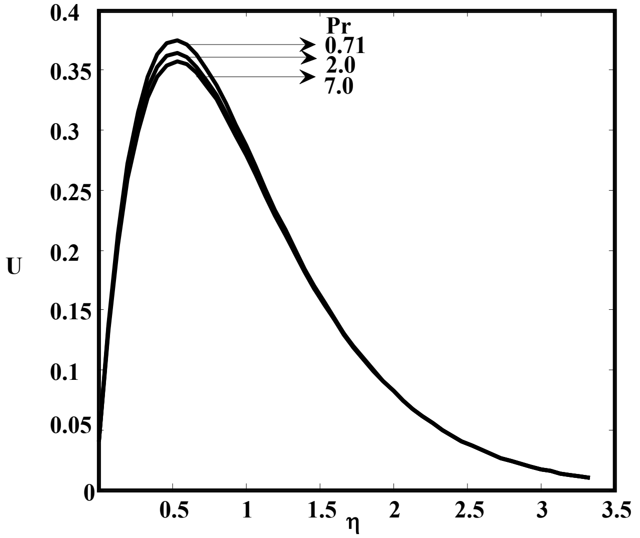

Figure.5. Velocity profiles for different values of Pr

Figure 6 shows that the effects of time t (t = 0.2, 0.4, 0.6) on the velocity profile in the presence of air (Pr = 0.71). It is observed that the velocity increases with enhancing values of t. The effect of the various values of the Schmidt number Sc (Sc = 0.16, 0.6, 2.01) on the velocity profile in the presence of air(Pr = 0 .71) at time t = 0.2 are shown in Figure 7. Here Sc measures the relative effectiveness of momentum and mass transport by diffusion. Further it is observed the momentum boundary layer decreases with increase in the value of Sc.

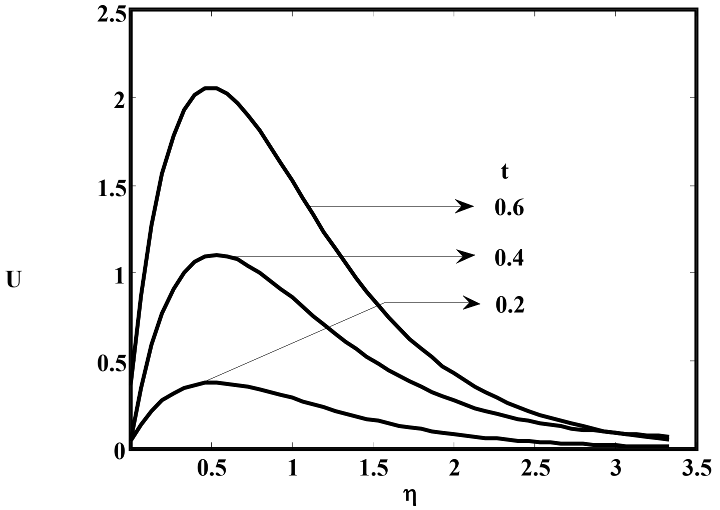

Figure.6. Velocity profiles for different values of t

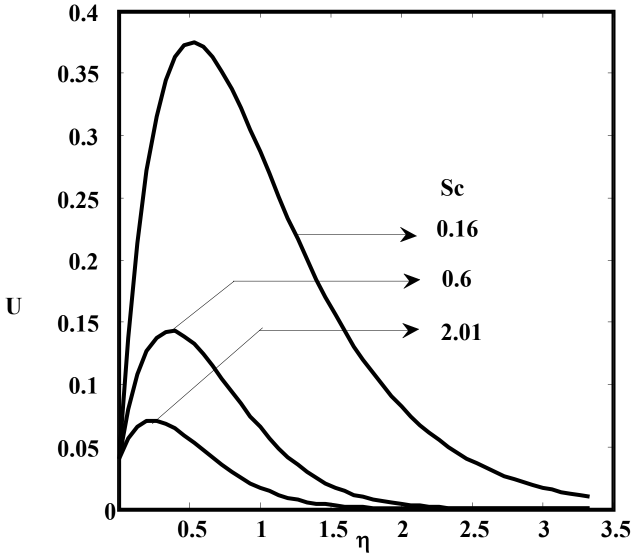

Figure.7. Velocity profiles for different values of Sc

In Figure 8, the effect of the velocity for different values of thermal Grashof number Gr and mass Grashof number Gc are shown graphically in the presence of air (Pr = 0.71) at time t = 0.2. It is observed that the velocity increases with increasing thermal Grashof number or mass Grashof number. This is due to the fact that buoyancy force enhances fluid velocity and increase in the value of Gr or Gc.

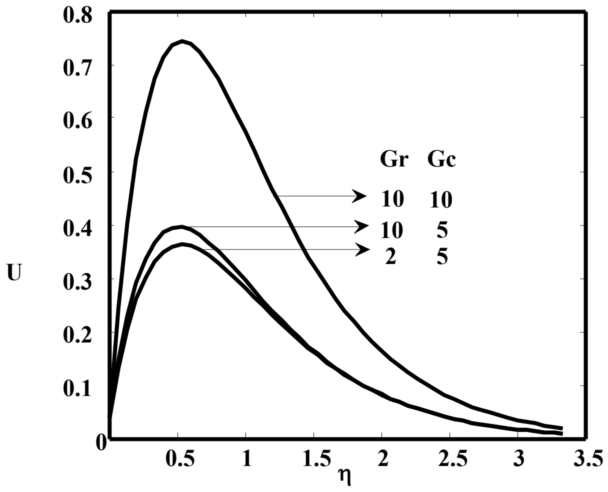

Figure.8. Velocity profiles for different values of Gr and Gc

Table 1 and Table 2 shows the effect of skin friction for the different values of thermal Grashof number Gr, mass Grashof number Gc, Prandtl number Pr, Schmidt number Sc, and time t. Table 1 shows the effect of skin friction in the presence of air (Pr = 0.71) and Table 2 shows the effect of skin friction in the presence of water (Pr = 7.0). In both the tables, we observe the skin friction increases with the increasing values of mass Grashof number and skin friction enhances with the increasing values of Schmidt number. When time t increases, the value of skin friction decreases.

Table 1 Skin friction profiles for air

t	Gr	Gc	Sc	τ
0.2	2	5	0.6	-0.665949
0.2	5	5	0.16	-1.833723
0.2	5	5	0.6	-0.775495
0.2	5	5	2.01	-0.339745
0.2	5	10	0.6	-1.502981
0.4	5	5	0.6	-1.590762
0.6	5	5	0.6	-2.431920

Table 2 Skin friction profiles for water

t	Gr	Gc	Sc	τ
0.2	2	5	0.6	-0.629829
0.2	5	5	0.16	-1.743424
0.2	5	5	0.6	-0.685196
0.2	5	5	2.01	-0.249447
0.2	5	10	0.6	-1.412682
0.4	5	5	0.6	-1.335357
0.6	5	5	0.6	-1.962712

V.CONCLUSION

A mathematical analysis has been presented of the transient free convection viscous flow past along a parabolic started infinite vertical plate with variable temperature which is induced by an uniform mass flux. The dimensionless governing equations are derived for the momentum, energy and mass conservation equations and the solution are obtained by laplace transform technique. The study has shown that as time t increases, the velocity increases as well as the concentration enhances and the temperature of the plate also increases. While the temperature as well as the velocity enhances leads to a reduces in Pr and the Schmidt number decreases with an enhancing values of concentration as well as the temperature of the plate. It is observed that the velocity of the plate increases with increasing values of thermal Grashof number Gr or mass Grashof number Gc. It is noticed that skin friction enhances with the increasing value of mass Grashof number Gc or Schmidt number Sc, but the trend is reversed in time t.

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